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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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A COMPARISON OF THE YERKES AND THE GREENWICH PHOTOGRAPHIC MAGNITUDES

By STURE HOLM[1]

ABSTRACT

A comparison has been made between the magnitudes of the *Yerkes Actinometries: First and Second Series*. The apparent differences between the two catalogues have been corrected for statistical errors. The remaining differences in the sense Second *minus* First Series are positive, indicating a scale correction of +0.20 for stars brighter than $6^{M}0$ and of $+^{M}03$ for stars fainter than $6^{M}0$.

The *Yerkes Actinometry: First Series* has also been compared with the Greenwich catalogues of photographic magnitudes. This has been done in order that the scale corrections derived could be compared with the scale corrections obtained by F. H. Seares. The present method yields results which are generally in good agreement with those of Mount Wilson. Lack of agreement in the case of late-type stars is attributed to the probability that the linear relation between scale correction and color index commonly employed may not be adequate for the purpose.

In his review[2] of the *Yerkes Actinometry: Second Series*, by A. S. Fairley (Y_2),[3] Eberhard has made a comparison between the magnitudes of this catalogue and those of the *Yerkes Actinometry* by Parkhurst (Y).[4] Eberhard computed the mean differences for 123 stars, common to the two catalogues, and obtained the following results (unit $= 0^{M}01$):

m........	<6.0	6.1–6.5	6.6–7.0	7.1–7.5	7.6–8.0	>8.0
Y_2-Y....	+23	+4	+7	+9	+2	−5

Eberhard followed the usual method of computing the mean differences for stars within equal intervals of magnitudes. Even if the

[1] Swedish Fellow of the American Scandinavian Foundation.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **66**, 321, 1931.

[3] *Astrophysical Journal*, **73**, 125, 1931. [4] *Ibid.*, **36**, 169, 1912.

two catalogues agree in scale value, this method will introduce a spurious difference, the *statistical error.* This error is due: (1) to the accidental errors of the magnitudes and (2) to the circumstance that the number of stars [illegible] ertain limits of magnitude is not generally constant but [illegible] on of magnitude.

The differences derive[illegible] Eberhard show a run with magnitude which indicates that they may be due, at least partially, to the statistical error. In his *Draft Report of Commission 25* (*Stellar Photometry*) of the International Astronomical Union for the meeting at Cambridge, Massachusetts, in September, 1932, Seares has quoted the foregoing figures as giving the definitive scale corrections to the magnitudes of the two catalogues. For this reason I have found it desirable to remove the statistical errors and to compute the real scale differences.

In his *Draft Report* Seares also gives the differences between several catalogues, including stars around the North Pole, in order to extend the North Polar Sequence. From the brief account of the methods used by Seares for computing those differences it is not clear whether the statistical error has been eliminated. For this reason I have also reconsidered the differences between Y and the Greenwich catalogues of photographic magnitudes.[5] Those catalogues will be treated as a whole and the common notation, Gr, assigned to them. If occasionally it proves necessary to distinguish between the two catalogues, the symbol Gr_1 will be used for the earlier catalogue and the symbol Gr_2 for the later one.

Methods for eliminating the statistical errors were first developed by Pannekoek in his work, "Researches on the Structure of the Universe."[6] Pannekoek, however, made certain assumptions concerning the form of the frequency curve of the magnitudes which, in effect, limit the applicability of his formulae to a few special cases. Formulae valid for any form of the frequency function have been developed by the author and will be used in the present article.[7]

[5] *Photographic Magnitudes of Stars Brighter than* $9^{M}.0$ *between Declination* $+75°$ *and the Pole Determined at the Royal Observatory, Greenwich, 1913; Photographic Magnitudes of Stars Brighter than* $9^{M}.0$ *between Declination* $+65°$ *and Declination* $+75°$ *Determined at the Royal Observatory, Greenwich, 1914.*

[6] *Publications of the Astronomical Institute of the University of Amsterdam,* 1, 1924.

[7] *Meddelande från Lunds Astronomiska Observatorium,* Ser. II, No. 59, 1931.

The magnitudes of the catalogue under examination will be denoted by m_1 and the dispersion of the magnitudes by μ_1. The catalogue with which it is compared will be called the "standard catalogue" and its magnitudes will be denoted by m. Let $\overline{m-m_1}$ signify the mean value of the differences between the two catalogues, and let $\bar{a}_1\ (m_1)$ signify the frequency function of the magnitudes of those stars of the catalogue under examination which also occur in the standard catalogue. Introducing the notation

$$\psi_1(m_1)=\log \bar{a}_1(m_1)\ ,$$

it can be proved that

$$\overline{m-m_1}=\mu_1^2\,\frac{d\psi_1(m_1)}{dm_1}\ . \tag{A}$$

This will be the apparent difference between the two catalogues if the zero points and scale values are the same. To get the true scale correction from the computed value of $\overline{m-m_1}$, this apparent mean difference must be corrected for statistical error. The correction to be applied is identical with the right-hand side of formula (A) with sign reversed.

Formula (A), however, is valid only for a single value of m_1. If we are to use the mean difference for stars within a wider interval, say from m_1+k to m_1 ($k>0$), the following formula may be used:

$$(\overline{m-m_1})_{m_1+k,\ m_1}=\mu_1^2\,\frac{d\psi_2(m_1)}{dm_1}\ . \tag{B}$$

Here

$$\psi_2(m_1)=\log\,[\bar{A}_1(m_1+k)-\bar{A}_1(m_1)]\ ,$$

and

$$\bar{A}_1(m_1)=\int_{-\infty}^{m_1}\bar{a}_1(m_1)dm_1\ .$$

Thus, $\bar{A}_1\ (m_1)$ signifies the total number of stars *brighter than* m_1 which occur in both catalogues.

Formula (B) is most suitable in case the number of stars is rather small, and therefore has been applied exclusively in the present in-

vestigation. Values of log $[\bar{A}_1(m_1+k)-\bar{A}_1(m_1)]$ have been computed for each tenth of a magnitude. These have been plotted and values of $\psi_2(m_1)$ have been read from the resulting curve. The values of the derivative of this function were computed in the following manner:

$$\frac{d\psi_2(m_1)}{dm_1}=\frac{\psi_2(m_1+.05)-\psi_2(m_1-.05)}{.10}.$$

An attempt was made to read off several values of $\psi_2(m_1)$ from the curve and to compute the derivative by means of an interpolation formula. The resulting derivative, however, differed by an insignificant amount from that obtained by means of the simple procedure described above.

The following groups of stars have been excluded from the comparisons: (1) close pairs for which Parkhurst gives only the combined magnitude, the other catalogue giving the magnitudes of the two stars separately; (2) stars for which Parkhurst gives the photographic magnitude only to the nearest tenth of a magnitude; (3) stars for which no subclass of spectral type is given in the *Henry Draper Catalogue;* and (4) stars having composite spectra. The symbols tabulated will be used.

Catalogue	Magnitude	Mean Error of 1 Mag.
Y	m_1	μ_1
Y_2	m_2	μ_2
Gr	m_3	μ_3

	Mean Error of 1 Difference
Y_2-Y	μ_{12}
$Y-Gr$	μ_{13}

COMPARISON BETWEEN THE "YERKES ACTINOMETRY: FIRST AND SECOND SERIES"

There are 131 stars common to the two catalogues and suitable for use in the comparison. The distribution of the stars in magnitude is given in Table I.

In Table I, 6.05 signifies the interval 6.00–6.09; 6.15, the interval 6.10–6.19, etc. From the data given in Table I, values of $\psi_2\ (m_1)$ and of $d\psi_2(m_1)/dm_1$ were obtained by the method outlined, and the values are listed in Table II.

In order to obtain the statistical correction it is also necessary to know the value of μ_1. Multiplying the probable error, which is given by Parkhurst for each star, by 1.483 yields the *internal* mean error, i.e., the mean error as computed from the residuals, individual observation *minus* adopted catalogue magnitude. We desire the *external* mean error, which is usually larger than the internal error.

TABLE I

m_1	N	m_1	N	m_1	N	m_1	N
4.45	1	....	....	....	....	....	....
5.05	....	6.05	2	7.05	2	8.05	2
.15	2	.15	1	.15	4	.15	6
.25	2	.25	3	.25	10	.25	5
.35	1	.35	2	.35	5	.35	4
.45	1	.45	3	.45	9	.45	2
.55	....	.55	3	.55	7	.55	....
.65	2	.65	5	.65	6	.65	1
.75	1	.75	3	.75	5	.75	....
.85	2	.85	2	.85	8	.85	....
5.95	2	6.95	7	7.95	9	8.95	1

TABLE II

m_1	$\psi_2(m_1)$	$\frac{d\psi_2(m_1)}{dm_1}$	m_1	$\psi_2(m_1)$	$\frac{d\psi_2(m_1)}{dm_1}$
4.95	1.510	....	6.95	3.370	....
5.00	....	+1.40	7.00	....	+1.30
5.05	1.650	....	7.05	3.500	....
5.45	1.970	....	7.45	3.615	....
5.50	....	+ .70	7.50	....	− .60
5.55	2.040	....	7.55	3.555	....
5.95	2.385	....	7.95	3.175	....
6.00	....	+ .95	8.00	....	−1.75
6.05	2.480	....	8.05	3.000	....
6.45	2.850	....			
6.50	....	+ .90			
6.55	2.940	....			

To obtain these external errors the stars were arranged according to m_1 and values of $\overline{m_2 - m_1}$ were computed for intervals of $0^M\!.5$. These values are to be found in column 3 of Table III. For each group the deviations of the individual magnitudes from the mean value were computed and from the squared values of all these residuals the

mean error of one difference was derived. This mean error was found to be $\mu_{12} = \pm^{\mathrm{M}}_{.}140$. According to the theory of errors, we have

$$\mu_1^2 + \mu_2^2 = \mu_{12}^2 \; , \tag{1}$$

where μ_1 and μ_2 are the *external* errors. The difficulty is to solve this indeterminate equation. I have proceeded according to the following reasoning:

It seems probable that the external error and the internal error are strongly correlated. I have therefore assumed that these two errors are in direct proportion. For this reason it is necessary to have the internal mean error for each catalogue. These errors will be denoted by μ_1', μ_2', and μ_3'.

Parkhurst and Fairley have each given the number of plates for each star and the probable errors, according to the number of plates. From this information the following mean values of μ_1' and μ_2' were derived:

$$\mu_1' = \pm^{\mathrm{M}}_{.}054 \; ; \qquad \mu_2' = \pm^{\mathrm{M}}_{.}049 \; .$$

Values of μ_1' and μ_2' were also computed for the several magnitude groups of Table III. μ_2' was found to be practically constant, but μ_1' increased slightly with magnitude. The differences, however, were so small that they could be neglected, and a constant value of μ_1' was used for all magnitudes.

Since the difference between μ_1' and μ_2' is very small, I have put $\mu_1 = \mu_2$. We then obtain, by means of (1),

$$\mu_1 = \mu_2 = \pm^{\mathrm{M}}_{.}099 \; .$$

Thus, the external mean error for the stars common to Y and Y_2 is about twice as large as the internal error.

The final results of the comparison between Y and Y_2 are given in Table III.

TABLE III

m_1	N	$\overline{m_2 - m_1}$	Corr.	Scale Difference	m_1	N	$\overline{m_2 - m_1}$	Corr.	Scale Difference
5.00–5.49 . . .	6	$+^{\mathrm{M}}_{.}253$	$-^{\mathrm{M}}_{.}014$	$+^{\mathrm{M}}_{.}24$	7.00–7.49 . .	30	$+^{\mathrm{M}}_{.}048$	$-^{\mathrm{M}}_{.}013$	$+^{\mathrm{M}}_{.}035$
5.50–5.99 . . .	7	.189	.007	.18	7.50–7.99 . .	35	+ .029	+ .006	.035
6.00–6.49 . . .	11	.028	.010	.02	8.00–8.49 . .	19	$-^{\mathrm{M}}_{.}005$	$+^{\mathrm{M}}_{.}018$	$+^{\mathrm{M}}_{.}01$
6.50–6.99 . . .	20	$+^{\mathrm{M}}_{.}046$	$-^{\mathrm{M}}_{.}009$	$+^{\mathrm{M}}_{.}04$	. . . .	. . . . .	. . . . .	. . . . .	. . . . .

Column 1 gives the magnitude groups according to Y. Stars with $m_1 < 5.0$ or $m_1 > 8.5$ were excluded. Column 2 gives the number of stars in each group and column 3 the mean apparent differences of magnitude. The fourth column gives the statistical correction, computed by multiplying the values of $d\psi_2(m_1)/dm_1$ of Table II by μ_1^2 and reversing the sign of the product. The last column contains the true scale differences $Y_2 - Y$.

Most of the values given in the third column of Table III differ from Eberhard's values mentioned on page 229. The discordances arise partly from the choice of stars for the comparison and partly because I have incorporated corrections to the magnitude of Y_2,[8] which were published after the appearance of Eberhard's review.

The last column of Table III exhibits a marked difference between the values of scale difference for magnitudes brighter than $6^{\mathrm{M}}.0$ and magnitudes fainter than $6^{\mathrm{M}}.0$. For magnitudes fainter than $6^{\mathrm{M}}.0$ it seems probable that the difference in scale value for $Y_2 - Y$ amounts to $+^{\mathrm{M}}.03$. For the brightest stars, however, the difference amounts to about $+^{\mathrm{M}}.20$. This value must be considered as uncertain since only thirteen stars are involved in its determination and a few rather large individual differences are present.

The magnitudes as determined by Fairley thus seem to be fainter than those determined by Parkhurst. It is difficult to see why the two catalogues should not agree since the observations were made at the same place, with the same instrument and reduced by the same method. Possibly the cause of the discrepancy may lie in some seasonal effect. Eberhard pointed out that there is a large variation of the differences with right ascension. Eberhard, however, divided the twenty-four hours into four parts of unequal width. The reason for such a division seems obscure, since the numbers of stars within the intervals are quite unequal. I have made the accompanying partition into four equal parts, chosen in such a way as to show as high a maximum and low a minimum as possible. A minimum appears near 18^{h} and a maximum near 0^{h}; both are probably due to some seasonal effect.

RA	3^{h}–9^{h}	9^{h}–15^{h}	15^{h}–21^{h}	21^{h}–3^{h}
N	37	20	24	50
$\overline{m_2 - m_1}$	+.04	+.08	−.11	+.12

[8] *Astrophysical Journal*, **75**, 427, 1932.

In order to investigate whether any color equation is present in the differences, stars with $m_1 < 6.0$ were corrected by $-^{M}_{.}20$ and stars with $m_1 > 6.0$ by $-^{M}_{.}03$. The stars were divided into three groups according to spectral type and the mean differences were computed, first for all stars and then for stars with $m_1 > 6.0$. The figures in brackets give the number of stars.

Sp.	5.0–8.5	6.0–8.5
B–A5	$^{M}_{.}00$ (55)	$+^{M}_{.}01$ (46)
A6–F9	$-.02$ (29)	$-.03$ (28)
G0–M	$+.02$ (46)	$+.01$ (43)

Stars of early and late spectral type show positive differences, while stars of intermediate type yield negative residuals. The number of stars within the group A6–F9, however, is so small that nothing can be said with certainty as to the reality of the difference.

TABLE IV

m_3	m_3-m_1		m_3	m_3-m_1	
	Δm	C		Δm	C
5.25	$+.22$	$-.11$	6.75	$-.11$	$-.07$
5.75	$+.12$	$.10$	7.25	$.15$	$.05$
6.25	$.00$	$-.10$	7.75	$-.16$	$-.06$

COMPARISON BETWEEN THE "YERKES ACTINOMETRY" AND THE GREENWICH PHOTOGRAPHIC CATALOGUES

I have chosen for investigation this pair of catalogues out of the many compared by Seares, for two reasons. The differences show a very peculiar run with magnitude, and the two catalogues have a large number of stars in common, making it possible to divide them into several groups according to spectral type and to compute the statistical corrections for each group separately. In this way it should be possible to determine whether or not the differences between the catalogues are dependent on both magnitude and color.

Seares starts from the assumption that the relation between the two catalogues may be expressed by means of the following formula:

$$m_1 - m_3 = \Delta m + cC \; , \qquad (2)$$

where Δm is the scale difference, C the color index, and c the "relative color coefficient." The results obtained by Seares are reproduced in Table IV.

For this new comparison the stars were arranged in four groups according to spectral type, namely B8–A2, A3–F5, F8–G5, K0–K5. It was intended to include approximately the same number of stars in each group. In order to narrow the range of color index for the first and for the last group, five stars of spectral types B2–B5 and 19 M-type stars were excluded. The distribution of the stars in magnitude is shown in Table V. It is noticeable that the bright stars are mostly of early type, while late spectral types dominate among the faint stars. This is due to the fact that Parkhurst included in his program only stars brighter than $7^{\mathrm{M}}.5$ visually.

TABLE V

m_3	B8–A2	A3–F5	F8–G5	K0–K5	m_3	B8–A2	A3–F5	F8–G5	K0–K5
4.45	2	......	......	......	7.15	6	8	......	2
.55	1	......	......	1	.25	11	12	5	6
.65	......	......	......	......	.35	10	12	8	2
.75	......	......	......	......	.45	6	5	6	10
.85	......	......	......	......	.55	5	6	11	14
4.95	......	......	......	......	.65	5	7	12	7
5.05	1	......	......	......	.75	4	7	7	10
.15	......	......	1	......	.85	2	3	7	7
.25	2	......	......	......	7.95	1	6	5	13
.35	2	......	1	......	8.05	......	4	13	13
.45	2	2	......	1	.15	1	3	5	5
.55	3	2	1	......	.25	......	......	2	19
.65	2	3	......	1	.35	......	1	2	9
.75	5	2	......	1	.45	......	......	2	9
.85	2	2	1	1	.55	......	......	5	6
5.95	1	......	1	2	.65	......	......	1	8
6.05	3	2	2	1	.75	......	......	2	3
.15	8	2	1	1	.85	......	......	1	1
.25	8	2	1	2	8.95	......	......	......	4
.35	6	4	1	1	9.05	......	......	......	2
.45	7	8	1	4	.15	......	......	......	3
.55	8	6	......	1	.25	......	......	......	......
.65	9	6	......	2	.35	......	......	......	......
.75	7	7	3	2	.45	......	......	......	......
.85	11	3	2	2	9.55	......	......	......	1
6.95	6	5	4	6					
7.05	11	15	5	6	All....	158	145	119	189

The procedure used for determining $\overline{m_1 - m_3}$ and μ_{13} was the same as that used before, with the exception that μ_{13} was derived for each separate magnitude, for reasons given below. The values of μ_{13} are given in Table VI, those of $\overline{m_1 - m_3}$ in Table IX.

The question arises as to how to separate μ_{13} into μ_1 and μ_3. On page 14 of Gr_1 the probable error for a single observation regardless of the magnitude is given as $\pm^{M}.064$. In Gr_2, page 13, the probable error is given as follows:

$$7.0\text{–}8.0\text{:p.e.} = \pm^{M}.080\,, \qquad 8.0-9.0\text{:p.e.} = \pm^{M}.069\,.$$

I have adopted the value $\pm^{M}.070$ as best representing the probable error of a magnitude determination from a single plate for the Greenwich stars included in this investigation. Since the number of plates

TABLE VI

m_3	μ_{13} B8–A2	μ_{13} A3–F5	μ_{13} F8–G5	μ_{13} K0–K5	μ_{13} B8–K5	m_3	μ_{13} B8–A2	μ_{13} A3–F5	μ_{13} F8–G5	μ_{13} K0–K5	μ_{13} B8–K5
5.00–5.49	± .195	......	......	......	± .163	7.50–7.99	± .155	± .121	± .117	± .119	± .130
5.50–5.99	.163	± .143	± .057	± .144	.161	8.00–8.49	......	± .267	.129	.144	.160
6.00–6.49	.104	.162	.059	.136	.139	8.50–8.99	......	......	± .128	.282	.250
6.50–6.99	.079	.123	.099	.121	.112	9.00–9.49	......	......	......	± .205	± .205
7.00–7.49	± .096	± .134	± .135	± .151	± .130						

is given for each star, probable errors may be computed for the individual stars, and the mean value of all such probable errors should constitute a fair determination of the average value of the internal mean error for one catalogue value.

To facilitate the computation of the average value of the mean error for the Yerkes stars, Parkhurst's Table II was used. From this table the probable error of a single determination was found to be about $\pm^{M}.065$. This value was used in the case of those Yerkes stars having only one determination of photographic magnitude. The probable errors for the different spectral subgroups within any given magnitude interval were found to differ by only small and accidental amounts, and the results are therefore listed according to magnitude as follows:

m_3	5.00–5.49	5.50–5.99	6.00–6.49	6.50–6.99	7.00–7.49
μ_1'	± .040	.038	.038	.040	.047
μ_3'	± .065	.074	.074	.071	.070

m_3	7.50–7.99	8.00–8.49	8.50–8.99	9.00–9.49
μ_1'	± .053	.058	.067	.074
μ_3'	± .071	.074	.074	.077

The value of μ_3' remains constant, owing to the fact that the number of observations is practically the same for all stars of *Gr*. The values of μ_1' show a steady increase with magnitude, since Parkhurst, on the average, obtained more observations of the brighter stars. The differences between the values of μ_1' for the faint and for the bright stars are considerably larger than the differences between the corresponding values for those stars common to *Y* and Y_2.

With the foregoing values of μ_1' and μ_3' as a basis, the following relations were selected for computing μ_1 and μ_3 from μ_{13}:

m_3............	5.00–6.99	7.00–8.49	8.50–9.49
	$\mu_3 = 2\mu_1$	$\mu_3 = \frac{4}{3}\mu_1$	$\mu_3 = \mu_1$

The values of μ_1 and μ_3 are given in Table VII. For the spectral groups A3–F5 and F8–G5 and for stars of the group Ko–K5 with $m_3 < 8.50$, the values of μ_3 are in good agreement, neglecting a few

TABLE VII

m_3	B8–A2		A3–F5		F8–G5		Ko–K5		B8–K5	
	μ_1	μ_3	μ_1	μ_3	μ_1	μ_3	μ_1	μ_3	μ_1	μ_3
5.00–5.49..	±.087	±.174	......	......	......	......	......	......	±.073	±.146
5.50–5.99..	.073	.146	±.064	±.128	±.025	±.051	±.064	±.129	.072	.144
6.00–6.49..	.047	.093	.072	.145	.026	.053	.061	.122	.062	.124
6.50–6.99..	.035	.071	.055	.110	.044	.089	.054	.108	.050	.100
7.00–7.49..	.058	.077	.080	.107	.081	.108	.091	.121	.078	.104
7.50–7.99..	±.093	±.124	.073	.097	.070	.093	.071	.095	.078	.104
8.00–8.49..	......	......	±.160	±.213	.077	.103	.086	.115	.096	.124
8.50–8.99..	......	......	......	......	±.091	±.091	.199	.199	.177	.177
9.00–9.49..	......	......	......	......	......	......	±.145	±.145	±.145	±.145
All....	±.057	±.094	±.076	±.118	±.070	±.095	±.092	±.121	±.080	±.119

of the most extreme magnitude groups where the values of μ_3 result from a very small number of stars. Stars of the group B8–A2, with $m_3 < 6.00$ and stars of the group Ko–K5 with $m_3 > 8.50$ have values of μ_3 decidedly above the average. Stars with $m_3 > 6.00$ of the group B8–A2, on the contrary, show values of μ_3 below the average. The adopted values of μ_3 are given in Table IX.

The average value of μ_1 for all stars was found to be $\pm^{\mathrm{M}}.080$. This may seem inconsistent with the value, $\mu_1 = \pm^{\mathrm{M}}.099$, found in the comparison, *Y* with Y_2, but the discrepancy is only apparent. The

value $\pm^{\mathrm{M}}.080$ applies to all stars of Y, while the value $\pm^{\mathrm{M}}.099$ is valid only for stars common to Y and Y_2. Such stars are all situated near the southern boundary of the polar cap observed by Parkhurst. For the whole catalogue of Y the number of observations for each

TABLE VIII

m_3	B8–A2		A3–F5		F8–G5		K0–K5	
	$\psi_2(m_3)$	$\frac{d\psi_2(m_3)}{dm_3}$	$\psi_2(m_3)$	$\frac{d\psi_2(m_3)}{dm_3}$	$\psi_2(m_3)$	$\frac{d\psi_2(m_3)}{dm_3}$	$\psi_2(m_3)$	$\frac{d\psi_2(m_3)}{dm_3}$
4.95	1.795		.355		.600			
5.00		+2.75		+7.40		+ .90		
5.05	2.070		1.095		.690			
5.45	2.580		2.085		1.060		1.485	
5.50		+ .70		−2.30		+1.95		+1.85
5.55	2.650		1.855		1.255		1.670	
5.95	3.360		2.580		1.795		2.060	
6.00		+1.40		+3.80		−1.50		+ .95
6.05	3.500		2.960		1.645		2.155	
6.45	3.720		3.410		2.020		2.485	
6.50		+ .20		+ .65		+2.95		+1.75
6.55	3.740		3.475		2.315		2.660	
6.95	3.740		3.970		3.090		3.225	
7.00		− .45		− .65		+1.70		+1.55
7.05	3.695		3.905		3.260		3.380	
7.45	2.880		3.395		3.775		3.870	
7.50		−2.90		−1.10		− .10		+ .55
7.55	2.590		3.285		3.765		3.925	
7.95			2.360		3.225		4.040	
8.00				−6.80		−2.30		− .80
8.05			1.680		2.995		3.960	
8.45					2.175		3.165	
8.50						−3.10		−2.20
8.55					1.865		2.945	
8.95							1.810	
9.00								−3.25
9.05							1.485	

photographic magnitude averages 4.7, whereas the average number of observations for stars in common with Y_2 is only 3.7. Using these figures, we may compute from the foregoing values of μ_1 the "external mean error" of one observation. We get from $Y-Gr$ the value $\pm.17$; and from Y_2-Y, $\pm.19$. These values are in good agreement,

TABLE IX

m_3	N	$\overline{m_1-m_3}$	μ_3^2	Corr.	Scale Diff.	C.I.	$\Delta m+cC$	N	$\overline{m_1-m_3}$	μ_3^2	Corr.	Scale Diff.	C.I.	$\Delta m+cC$
	B8–A2							A3–F5						
5.00–5.49	7	−^{M}.173	.025	−^{M}.069	−^{M}.24	+^{M}.003	−^{M}.22	2	−^{M}.155	.012	−^{M}.090	−^{M}.24	+^{M}.285	−^{M}.19
5.50–5.99	13	−.100	.025	−.018	−.12	.000	−.12	9	−.216	.012	+.028	−.19	.178	−.10
6.00–6.49	32	+.011	.008	−.011	.00	+.004	.00	18	+.053	.012	−.046	+.01	.167	+.02
6.50–6.99	41	+.096	.008	−.002	+.09	−.002	+.11	27	+.124	.012	−.008	+.12	.206	+.12
7.00–7.49	44	+.198	.008	+.004	+.20	+.010	+.15	52	+.189	.012	+.008	+.20	.214	+.16
7.50–7.99	17	+.127	.008	+.023	+.15	+.002	+.16	29	+.179	.012	+.013	+.19	.258	+.18
8.00–8.49	1	......	......	......	......	......	......	8	+.093	.012	+.082	+.18	+.290	......
	F8–G5							K0–K5						
5.00–5.49	2	−.195	.012	−.011	−.21	+.780	−.13	1	......	......	......	......	......	......
5.50–5.99	3	−.057	.012	−.023	−.08	.560	−.06	5	+.058	.012	−.022	+.04	+1.188	.00
6.00–6.49	6	+.052	.012	+.018	+.07	.740	+.07	9	+.193	.012	−.011	+.18	1.143	+.11
6.50–6.99	9	+.170	.012	−.035	+.14	.670	+.16	13	+.223	.012	−.021	+.20	1.105	.19
7.00–7.49	24	+.219	.012	−.020	+.20	.667	+.18	26	+.263	.012	−.019	+.24	1.105	.21
7.50–7.99	42	+.205	.012	+.001	+.21	.638	+.20	51	+.253	.012	−.007	+.25	1.122	+.23
8.00–8.49	24	+.173	.012	+.028	+.20	.689	......	55	+.221	.012	+.010	+.23	1.125	......
8.50–8.99	9	+.060	.012	+.037	+.10	+.753	......	22	−.020	.025	+.055	+.04	1.154	......
9.00–9.49	......	......	......	......	......	......	......	5	−.274	.025	+.081	−.19	+1.254	......

especially if we consider that the mean error also has its own mean error.

From the values of Table V, values of $d\psi_2(m_3)/dm_3$ were determined by the same method used for determining the corresponding function of m_1. The values of $\psi_2(m_3)$ and of its computed derivative are given in Table VIII.

The results of the comparison are given in Table IX. The first five columns for each spectral group are self-explanatory. Column 6 gives the mean color index for the stars. Using this value and the values of Table IV, the scale corrections according to Seares have been computed from formula (2) and are reproduced in column 7. The color indices used are the Mount Wilson values given by Seares in his *Draft Report* and are reprinted here.

Spectrum........	B8	A0	A2	A5	F0	F5	G0
	−.06	.00	+.04	.12	.22	.35	.54
Spectrum........	G5	G8	K0	K2	K5	M	
	+.78	.95	1.02	1.20	1.40	+1.65	

The values of C for spectral types not included in this list have been graphically interpolated.

Comparison of column 5 with column 7 for the first three spectral groups shows that good agreement exists between the scale corrections derived here and those obtained by Seares. For the group K0–K5, however, the values in column 7 are smaller throughout than those of column 5. The method used by Seares thus gives a good representation of the scale differences, except for stars with large color index. Apparently a linear relation between scale correction and color index does not give a sufficiently accurate representation of the true relation.

LICK OBSERVATORY
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ASTROMETRY WITH MIRRORS AND LENSES[1]

By FRANK E. ROSS

ABSTRACT

As part of the development work for the proposed 200-inch reflecting telescope of the California Institute of Technology, a lens has been designed by the writer, to be placed near the focus of the mirror with the object of correcting the large outstanding aberration of coma, which is serious in a mirror of the high speed ratio proposed. A trial lens has been made and applied to the Mount Wilson 60-inch telescope. The present paper is concerned primarily with the astrometric characteristics of the correcting lens and mirror system, as compared with those of the usual mirror systems, but has developed into a comparative study of all the telescopes available, ten in all.

The two types of astrometric problems are outlined: (*a*) that in which errors of differential co-ordinates are of importance; (*b*) that in which changes in these errors, from epoch to epoch, are of importance. Problem (*b*) has been given the more attention, because of its bearing on the determination of the parallaxes and relative proper motions of stars. It is shown that the equalization of magnitudes customarily employed in the determination of relative positions is not so essential as is usually considered to be the case and that accurate relative positions of a field of stars ranging widely in magnitude and in relative position can be obtained with the instruments now used in astrometric problems. It is demonstrated that the correcting lens greatly enlarges the field over which accurate positions can be obtained; for example, with a field of stars 10′ from the axis the average error in position is reduced by three-fourths (Table VI*a*). It is also demonstrated that great accuracy may be obtained in the measurement of star images with a diameter as large as 0.90 mm, and that the relative positions in a field of stars ranging over 6 mag. in brightness may be precisely determined—a matter of the greatest importance in connecting the Boss system of proper motions with the motions of faint stars and spiral nebulae. The *R–L* discordance in the measurement of star images has been given some attention. An attempt to correlate it with space perception gave no correlation and indicates that bisecting a star image is probably a matter of equating magnitudes.

I. INTRODUCTION

The present inquiry is concerned with the relative accuracy of measurement of the positions of the stars obtained with reflecting and refracting telescopes of various sizes and focal lengths. It owes its inception to the addition of a correcting lens of zero power to the equipment of the 60-inch reflecting telescope at Mount Wilson. This lens is experimental, designed by the writer on the initiative of Dr. G. E. Hale and made by the J. W. Fecker Company with the idea that, if successful, one of similar design would be used as auxiliary equipment with the proposed 200-inch telescope of the California Institute of Technology.

It is well known that the only outstanding optical error of any

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 467.

importance in the Newtonian parabolic mirror is coma. The magnitude of this aberration increases linearly from the center of the field and as the square of the aperture ratio of the mirror. In the case of the 200-inch mirror the effect of the aperture ratio would be the more troublesome of the two factors, for an amount of coma which is not serious in a mirror of aperture ratio 6, say, would be magnified four times in a mirror of ratio 3. In the case of the Bergedorf reflector of 1 m aperture and 3 m focal length, coma is perceptible, under the best conditions of seeing, at a distance of only $2'$ from the axis. Since the proposed 200-inch mirror will have an aperture ratio of 3.3, and since the linear scale will at the same time be great, the effect of coma, in many investigations, will be serious.

The problem of the elimination of coma from a mirror system has been solved mathematically by Schwarzschild by using two mirrors, each of special form, combined in a Gregorian system. Another solution of the problem, a corollary of Schwarzschild's solution, has been worked out by H. Chrétien,[2] in which the two specially surfaced mirrors are combined as in the ordinary Cassegrain telescope, thus giving the advantage of a short tube length, a distinct gain over the Gregorian type devised by Schwarzschild. Chrétien's arrangement has, however, the disadvantages of a relatively large F number, which means low speed for surface brightness, and, probably more important, of a curved field, necessitating the use of curved plates. A telescope of 40-inch aperture of the Chrétien type is now being constructed by G. W. Ritchey for the United States Naval Observatory, having F number 6.8,[3] and one of Schwarzschild's type has been made and is now being installed by Professor W. A. Cogshall at the Observatory of the University of Indiana. These two telescopes will no doubt prove useful in special problems. For the elimination of coma, a correcting lens of the type now under investigation, if used in the axis without the Newtonian flat, has the advantage of being slightly more economical of light, since the absorption is somewhat less than the loss of light with a silvered mirror in average condition. The actual loss with such a lens is about 0.25 mag., a trifling amount in the mensurational and photometric problems to which the lens is adapt-

[2] *Revue d'optique*, **1**, 13 and 49, 1922. [3] *Scientific Monthly*, **33**, 283, 1931.

ed. In view of the many lines of investigation to which a large reflecting telescope such as the proposed 200-inch must be put, there is no doubt that it will be found difficult to improve upon the present Newtonian and Cassegrain systems, when provided with a correcting lens inserted near the focal plane for the elimination of coma.

As already stated, the present paper is primarily concerned with the investigation of the mensurational or astrometric characteristics of the correcting lens, which are of special significance in one of the two important lines of research to which this type of lens is adapted and to which it owes its inception. It was early realized, however, that the usefulness of the investigation would be greatly increased if it were extended to include other telescopes and optical systems now being extensively used in astrometric problems. Ten different optical systems were examined:

Mount Wilson Observatory	Yerkes Observatory
100-in. Cassegrain	40-in. refractor
100-in. Newtonian	24-in. reflector
60-in. Cassegrain	24-in. reflector and correcting lens
60-in. Newtonian	10-in. Bruce
60-in. Newtonian and correcting lens	3-in. Ross

Any investigation of the precision of photographic measurement should cover the two important cases in which astronomy is interested. The first of these may be referred to as the determination of absolute position, in the restricted sense of position relative to chosen reference stars. Examples are measures of the positions of comets, planets, and asteroids. The second and more important type of investigation includes, for example, determinations of stellar parallax and relative proper motion, in which errors in the so-called absolute positions are of no consequence. In these cases it is important only to keep the errors in position, if such exist, constant from plate to plate and from epoch to epoch—a problem which has engaged the close attention of astronomers ever since the photographic plate has been employed for stellar parallax. Briefly, the precautions necessary in precise astrometry are equalization of magnitude, as far as possible, and of the effective exposure on each plate of the series; maintenance of constant conditions of development and of a constant position of the field with respect to the optical axis of the lens or

mirror; and, what is difficult to attain, the making of all exposures under the same seeing conditions. Since it is not possible to adhere rigorously to these conditions, variations in the position of the object being investigated occur from plate to plate, and it is important to note that it is these variations, or the variable part of the error itself, which are significant in the second case. In addition to these recognized causes of this troublesome dispersion might be mentioned the slight changes in focal setting which produce a change in the form of the image on the plate. Many optical defects, of both mirrors and lenses, cause this dependence on focal setting, the most important, in the case of mirrors, being coma and astigmatism. It is of course true that the equalization of the magnitudes of the stars measured, as generally adopted in parallax determinations, does away with a large part of the error. But in the equally important determination of relative proper motions, where stars ranging widely in magnitude must be compared, such a procedure is not practicable.

2. THE CORRECTING LENS

The correcting lens of zero power eliminating the outstanding coma of mirrors will be described in an early paper, which will give a fairly complete theory of the simplest or two-component type. It will also be shown that the solution is not unique and that a series of lenses may be designed for correcting coma. Such a correction, however, introduces other aberrations, for example, chromatic and spherical aberration, astigmatism and distortion. Fortunately these are small, and proper design can do much to reduce one or more of them, depending upon the particular requirements, for example, a moderate or extensive field; refined color correction; in particular, color magnification. One of the most troublesome of these small residual aberrations to deal with is spherical aberration. It is hoped that this can be reduced by zonal retouching, a method employed extensively by the manufacturer for lenses of all types. This method has not been used with the present lens, however, since the usual procedure of retouching is not applicable to a lens which must function in a highly convergent beam. An auto-collimating method of making the desired correction has therefore been developed, in which the plane mirror is replaced by a spherical mirror of short radius.

PLATE V

MESSIER 8 PHOTOGRAPHED WITH THE 60-INCH NEWTONIAN REFLECTOR AND CORRECTING LENS

Exposure, 150 minutes on Imperial Eclipse plate. Field, 81′×63′; 1 cm=4′.9

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PLATE VI

DUMB-BELL NEBULA PHOTOGRAPHED WITH THE 60-INCH NEWTONIAN REFLECTOR AND CORRECTING LENS

Exposure, 60 minutes on Imperial Eclipse plate. Enlargement, five times; 1 cm = 55″

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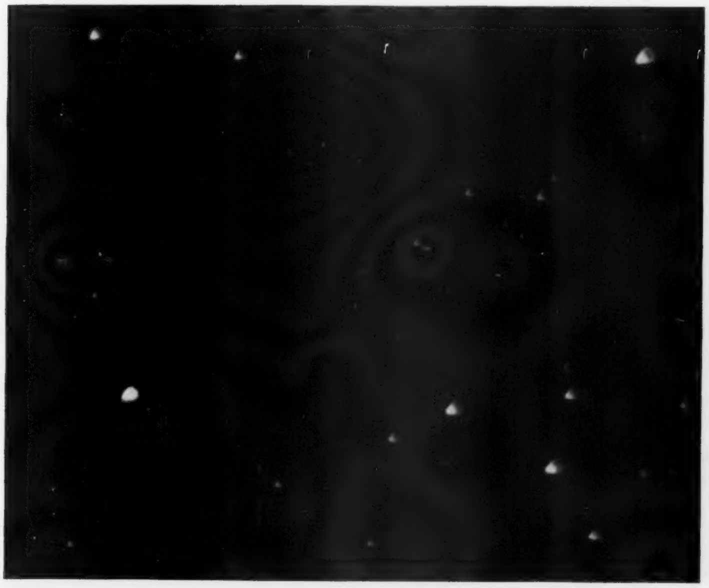
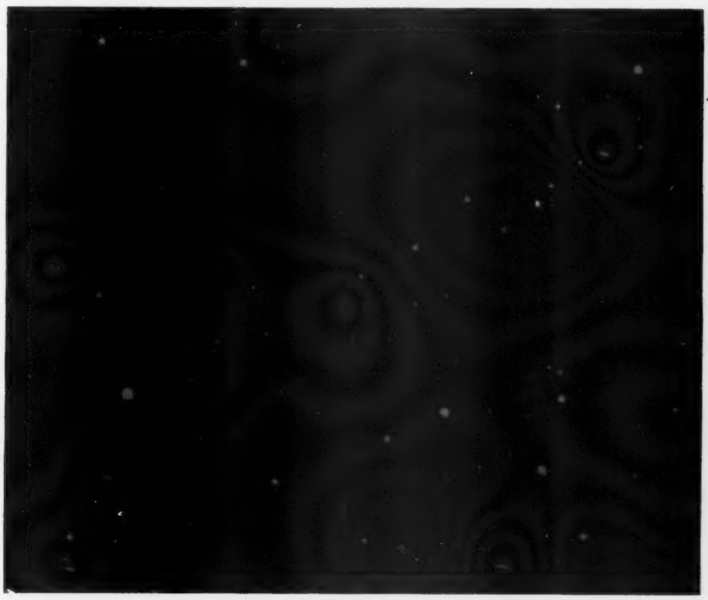
PLATE VII





Fivefold Enlargement of a Field Centered 27′ from the Axis of the 60-Inch Newtonian Reflector, Photographed without (Above) and with (Below) Correcting Lens

Scale, 1 cm = 55″

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The necessary installation is now being prepared by Mr. Dalton of the Mount Wilson Observatory optical shop, who will thus refigure both the present corrector lens and the one he is now constructing for use with the 100-inch reflecting telescope.

The field of any Newtonian mirror curves slightly inward with astigmatism,[4] which, however, is always masked by coma. Coma is eliminated by the corrector at the expense of astigmatism, which increases as the square of the distance from the axis. The amount of astigmatism present depends upon the design of the corrector and can be made greater or less, depending on the relative importance of the various outstanding aberrations mentioned above. In the case of the corrector lens with which we are now concerned, the astigmatism first becomes noticeable at about 55′ from the axis and is not objectionable at 60′. From this point on, it increases rapidly. In order to demonstrate the performance of the lens, three illustrations have been prepared. Plate V is a reproduction of a $6\frac{1}{2}\times 8\frac{1}{2}$-inch plate of Messier 8 taken with the correcting lens at the 60-inch Newtonian focus by N. U. Mayall and the writer, exposure time 150 minutes. Plate VI is a fivefold enlargement of a negative of the Dumb-bell Nebula taken by the writer with the same combination and an exposure time of 60 minutes. Plate VII shows enlargements of the same field taken without and with correcting lens, the field in both cases being situated at a mean distance of 27′ from the axis (NW. of the field measured in N.G.C. 6633).

The lens is only approximately of zero power, as it increases the focal length of the mirror by 4.4 inches, or 1.4 per cent. It is of 8-inch aperture, is placed 15 inches from the Newtonian focus, and displaces the focal point outward 2.7 inches.

3. PROGRAM

An open star cluster including stars of a great range in magnitude appears ideal for the investigation. Cluster N.G.C. 6633 was chosen. The measures were limited to only a small part of the cluster, twenty stars in all, ranging in magnitude from 8.4 to 14.8 and contained within a rectangle $7\overset{\prime}{.}1$ in α by $5\overset{\prime}{.}4$ in δ. In general, two series of

[4] Not to be confused with the astigmatism of a mirror distorted by temperature changes or mechanical stresses.

plates were obtained with each instrument: one with the measured field central and the other with the field displaced toward the edge of the plate. Several exposures were made in each position, on the same and on different nights. In some cases, two exposures were made in succession on the same plate, counting as two plates.

Rectangular co-ordinates were measured by the writer on the Gaertner machine regularly used at the Yerkes Observatory for parallax measures. There has been no recent determination of the screw or bearing errors of this instrument. An investigation by Dr. O. J. Lee and Dr. Hannah B. Steele in 1917[5] showed that the errors of the screw were small. With the method of reduction adopted for the present investigation, any outstanding errors may be considered eliminated, for in large part they combine with the scale value, which is automatically eliminated. This assumes, of course, that there are no sudden irregularities, which is highly improbable. The bearing errors, which enter as an inequality of a single turn, are perhaps more serious. Their effect was reduced to a minimum by the following procedure: After each run across the field, only one bisection of each image being made, the plate was shifted in position by one-half of one turn of the measuring screw, or 0.5 mm, and then remeasured. This was repeated for the reversed position of the plate. Each plate was thus measured four times, and each distance therefore depends upon eight readings of the drum, well distributed over its circumference. The bearing errors are therefore practically eliminated. In measuring declinations, however, the plate could not be shifted, owing to the construction of the machine, and the elimination is not so thorough.

The reduction of the measures was by the dependence method,[6] which is admirably adapted to the investigation. Three of the brightest stars, forming a well-conditioned triangle, were selected as reference stars within the area chosen for measurement. Their positions, taken from the A. G. Leipzig *Catalogue*, are given in Table I. Nominal errors in these positions, used as a base, of course do not affect the investigation.

[5] *Publications of the Yerkes Observatory*, **4**, 65, 1917.

[6] F. Schlesinger, *Astronomical Journal*, **37**, 77, 1927.

4. DETAILS OF OBSERVATIONS

I am greatly indebted to Dr. A. van Maanen for taking all the plates exposed at the 100-inch Newtonian and Cassegrain foci and at the 60-inch Cassegrain. The seeing for these exposures averaged 2 to 3 on a scale of 10. Eastman 40 plates were used and developed in a soft-working metol developer. The exposure time for the Cassegrain plates was 15 minutes, for the Newtonian, 5 minutes. Contrary to expectation, the 100-inch Cassegrain plates showed fainter stars than the 60-inch Cassegrain and a limiting diameter of $0''.75$, which is smaller than that for the same mirror in Newtonian form,[7] despite the comparatively poor seeing conditions. All the plates were taken during the summer of 1931, except those with the 100-inch Cassegrain, which were taken in 1932.

TABLE I

POSITIONS OF REFERENCE STARS

Star	B.D. No.	A.G. No.	α 1875	δ 1875	Mag.
A.	$6^\circ 3788$	8529	$18^h 21^m 49^s.670$	$+6^\circ 31' 28''.70$	8.6
B.	3782	8525	21 33.890	27 26.90	8.5
C.	3778	8522	21 24.130	30 53.90	9.0

Most of the exposures at the 60-inch Newtonian telescope were made by the writer, a few being taken by Dr. Hubble. Plates, exposures, and developing conditions varied to such an extent that I feel that full justice has not been done to the capabilities of this instrument. Perhaps the most serious handicap to uniformity was the prevailing astigmatism of the mirror, which made the focal setting uncertain and variable and led to an asymmetry of image seriously affecting the precision. The results for this instrument given in Tables IV–VI are therefore subject to considerable improvement.

I am indebted to Dr. G. W. Moffitt for the series taken with the Yerkes 40-inch refractor. The plates used were Eastman Wratten and Wainwright panchromatic, exposed behind the yellow filter (Schott G7), regularly used since March, 1931, for the routine parallax program of the Yerkes Observatory. The exposure time

[7] E. Hubble, *Mt. Wilson Contr.*, No. 453; *Astrophysical Journal*, **76**, 109, 1932.

was 15 minutes, which was sufficient only for the measurement of stars to magnitude 13.8 photographic, leaving eight stars of the series unmeasurable. The seeing varied from 2 to 3. Usually, with seeing 3 or better, the star images obtained with this instrument are triangular in form and therefore difficult to measure. These conditions occurred in the present series, in which four of the plates exposed under better conditions nevertheless show relatively large residuals.

The plates with the Yerkes 24-inch reflector without corrector were taken by Dr. G. Van Biesbroeck and the writer; those with the corrector, by the writer. The exposure time was 5 minutes. The exposures with the Bruce 10-inch and the Ross 3-inch were made by the writer, exposure time 60 minutes.

TABLE II

SCALE VALUE AND FOCAL LENGTH

Instrument	Scale (per mm)	Focal Length	
100-in. Cass.*	$5''.0236$	$41^{m}.059$	$1616^{in}.5$
100-in. Cass.†	5.0166	41.117	1618.8
100-in. Newt.	15.973	12.913	508.40
60-in. Cass.	8.223	25.083	987.5
60-in. Newt.	27.080	7.617	299.87
60-in. Z.C.	26.66	7.737	304.6
40-in. Refr.	10.653	19.362‡	762.3
24-in. Refl.	87.33	2.362	92.99
24-in. Z.C.	82.64	2.496	98.27
10-in. Bruce	160.4	1.286	50.62
3-in. Ross	386.8	0.533	21.00

* Value of August 7, 1932 ($t=23^\circ.3$).
† Value of August 25, 1932 ($t=25^\circ.2$).
‡ E. E. Barnard's value is 19.354 m (visual).

5. SCALE VALUE

On account of the method of reduction employed, the exact scale value is of no importance. In addition to the exposures on N.G.C. 6633, Dr. van Maanen made some with the 60-inch Cassegrain on the open cluster Melotte 179 which I have used in order to strengthen the scale value. Three pairs of stars in each cluster whose positions occur in the A.G. Leipzig *Catalogue* were chosen for measurement, the average separation being 1200″. From these six pairs the mean scale value for the 60-inch Cassegrain was found to be $8''.223$, with a probable error of $0''.0025$. With this value of the scale the

separation of two faint stars, p and a star of the same magnitude called x, was measured on all the 60-inch Cassegrain plates and found to be 538.″24. Star x is west of p by this amount and 1.″70 N. The scale value of all the other telescopes was determined by measuring the distance px on several plates of each series. The difference in the values for the 100-inch Cassegrain for the two nights on

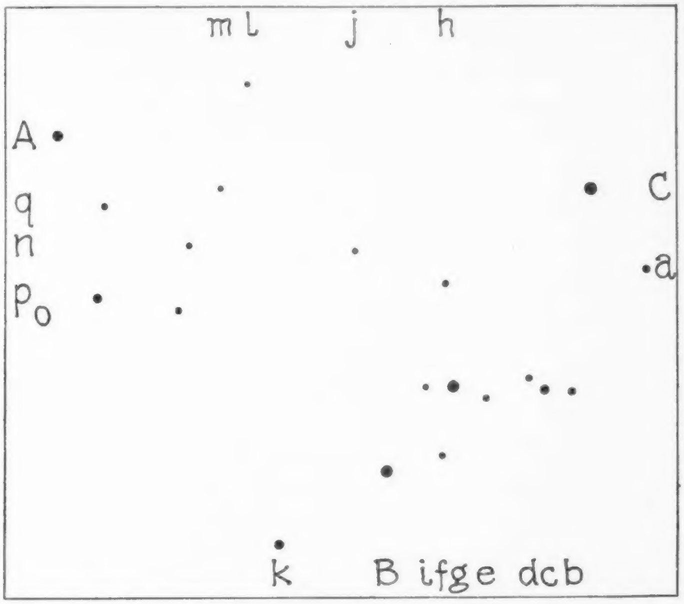

FIG. 1.—Field of stars in N.G.C. 6633 used for astrometric tests

which plates were obtained (Table II) is probably due to a change in the position of the plate-holder with respect to the tube of the telescope.[8]

6. ADOPTED MEAN POSITIONS

The stars chosen for measurement are plotted in Figure 1. Their positions and photographic magnitudes are in Table III. The magnitudes were obtained from a polar comparison made with the Yerkes 24-inch reflector. The right ascensions are the unweighted

[8] I have found by computation that a movement of the secondary mirror inward by 1 mm decreases the focal length by 50 mm, the focal point at the same time moving inward by 9 mm. It would therefore require a change of 11 mm in the plane of the photographic plate to produce the observed change in focal length.

means of measures on the Mount Wilson 60- and 100-inch Cassegrain plates taken with the field central. The declinations are the mean result of the nine photographs with the 100-inch Cassegrain, the 60-inch Cassegrain plates being omitted for the following reason: For the discussion and comparison which follows, the standard positions, which are given in Table III, should be as free as possible from accidental and systematic errors. During the observations the 60-inch mirror suffered a slight astigmatism in declination, affecting positions in this co-ordinate, for which the average difference between the two instruments is 50 per cent greater than in right ascension. The 100-inch plates were taken after the remounting of the mirror,[9] which now gives practically perfect images. Although the 100-inch Newtonian measures are surprisingly accurate, they have not been included in the mean result because of the possibility of their being slightly affected by coma, which is ten times greater for the Newtonian than for the Cassegrain combination.

TABLE III

MAGNITUDES AND ADOPTED MEAN POSITIONS FOR 1900.0

Star	Mag.	Comp. Stars A, B, C		Comp. Stars *k*, *p*, *a*	
		α	δ	α	δ
g	8.4	$18^h21^m30^s.689$	$+6°28'30''.90$	......	......
c	9.3	26.140	28 24.85	......	......
k	10.9	39.019	26 35.99	......	......
i	11.6	32.418	28 32.35	$^s.419$	$''.34$
b	11.8	23.914	28 22.56	.915	.56
d	12.3	27.167	28 41.07	.167	.08
p	12.7	47.842	29 30.85	......	......
a	12.9	21.384	29 51.96	......	......
f	13.6	31.108	27 36.39	.107	.37
o	13.8	43.828	29 22.36	.828	.34
n	13.9	43.504	30 7.65	.502	.64
m	14.0	41.740	30 55.40	.740	.39
e	14.2	28.989	28 21.30	.990	.30
l	14.2	40.511	32 6.14	......	......
h	14.4	31.361	29 44.18	.362	.17
q	14.6	47.570	30 36.40	.570	.40
j	14.8	35.595	30 5.01	.593	.97

[9] F. G. Pease, *Publications of the Astronomical Society of the Pacific*, **44**, 308, 1932.

7. MEAN RESULTS FROM CENTERED PLATES

The total range in magnitude of the stars chosen for measurement is 6.4 (Table III), while the mean magnitude of the comparison stars is 8.7 (Table I). The determination of their relative positions is therefore a severe, but very important, test, because it holds out the possibility of directly connecting the faint stars and distant spirals with the Boss system or any other system of positions and proper motions which may finally be adopted. From other points of view, however, it is of interest and importance also to adopt a set of fainter comparison stars. To this end I have chosen stars *a*, *k*, and *p*, of mean magnitude 12.2. The adopted positions obtained from these comparison stars are given in the last two columns of Table III. In general, therefore, the results and conclusions hold for (*a*) positions of faint stars based on A.G. comparison stars; (*b*) positions of stars based on comparison stars of the magnitude of those in the *Astrographic Catalogue*. Tables IV*a* and IV*b* indicate the position for each star, expressed as a correction to be added to the mean position in Table III, obtained by using comparison stars A, B, and C, and *k*, *p*, and *a*, respectively, and all the centered plates taken with each of the ten instruments. The number of plates is given in the second column; the mean residual, without regard to size, in the last column. The stars are arranged in the order of decreasing brightness.

It is important to know for each instrument and for each star the dispersion of the individual values. These results are given in Tables V*a* and V*b*. The data in Tables IV are of significance in the determination of absolute positions in the sense defined on page 245, while Tables V are of significance in the determination of proper motions and relative parallaxes, a matter of much greater importance.

Study of Tables IV.—The 100-inch and 60-inch Cassegrain results are in close agreement. If the three brightest stars, *g*, *c*, and *k*, be excepted, the mean difference in positions obtained with bright comparison stars is only $0^s.0023$ in α and $0''.048$ in δ, the latter affected by astigmatism as noted above. With faint comparison stars these differences become $0^s.0016$ and $0''.036$, respectively, only slightly smaller, one of many indications of a very small magnitude equation for these two telescopes, which is a very gratifying result indeed.

Comparison of the 100-inch Cassegrain and Newtonian results

TABLE IV*a*

MEAN POSITIONS FROM CENTERED PLATES, COMPARISON STARS A, B, C

Instr.	No. Pl.	*g*	*c*	*k*	*i*	*b*	*d*	*p*	*a*	*f*	*o*	*n*	*m*	*e*	*l*	*h*	*q*	*j*	Mean ±
									Right Ascension, Unit = 0ˢ.001										
100-in. Cass.	9	−2	+2	+2	0	+1	+1	0	+1	+2	+1	+2	0	−1	0	−1	−1	+2	0ˢ.0011
60-in. Cass.	4	+2	−2	−3	−1	0	−1	−1	−1	−1	−2	−1	0	+1	0	0	+2	−2	.0011
100-in. Newt.	4	+3	+4	+8	+3	+6	+8	0	+3	+5	0	0	0	+3	−1	+1	−3	+3	.0030
60-in. Newt.	5	0	+1	+2	+2	+6	+3	+4	−4	−2	+5	+5	+5	0	+3	−2	+7	+2	.0031
60-in. Z.C.	5	+1	0	+8	+8	+3	+7	+1	+5	+10	+2	+1	+4	+10	0	+3	−2	+6	.0042
40-in. Refr.	6	+4	+1	+1	−2	+2	−1	−4	−3	.....	−8	.....	.....	.....	.....	.....	.....	.....	.0029
24-in. Refl.	4	+3	+1	+8	+3	−19	−6	+10	−2	+14	+21	+41	+31	+1	+35	+11	+22	+17	.0144
24-in. Z.C.	3	−5	+5	+8	+25	+12	+33	−7	+31	+32	+22	+13	+14	+20	+39	+27	+7	+21	.0186
10-in. Bruce	3	−21	−20	−21	−32	−16	−3	−8	−36	−32	−22	−36	−20	.....	.....	.....	.....	.....	.0220
3-in. Ross	3	−21	−50	−41	+8	−16	−43	−8	−46	−52	+8	−36	−20	.....	.....	.....	.....	.....	0.0300
									Declination, Unit = 0″.01										
100-in. Cass.	9	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
60-in. Cass.	4	−2	+4	+9	+8	+10	+6	+8	+3	+9	+7	+2	0	+6	+1	−4	+3	0	0″.048
100-in. Newt.	4	−2	−4	−1	−2	−2	−1	−1	−5	−1	+7	−3	−4	−3	+2	−4	−3	+1	.026
60-in. Newt.	5	+2	+6	−9	−5	+1	−5	−2	+3	+1	−1	+8	+9	+4	+20	+6	+7	10	.058
60-in. Z.C.	5	+7	−1	−1	−3	−2	−6	+4	−7	+3	+7	+1	−3	+3	−3	+3	+5	1	.035
40-in. Refr.	6	+3	+1	−6	−6	−2	−5	−19	−14	.....	−19	.....	.....	.....	.....	.....	.....	.....	.083
24-in. Refl.	4	−10	−5	−2	+17	+15	+48	+96	+87	+93	+106	+119	+135	+57	+165	+118	+150	101	.78
24-in. Z.C.	3	+5	+4	+25	+18	+27	+55	+35	−6	+104	+35	+43	+32	+40	+14	+47	+35	+63	.35
10-in. Bruce	3	−10	+5	+39	+15	+36	+47	+5	−4	+51	+56	+5	−20	.....	.....	.....	.....	.....	.22
3-in. Ross	3	+10	+15	−41	+65	−4	−23	−25	−104	−81	−74	−170	−130	.....	.....	.....	.....	.....	0.61

TABLE IV*b*

MEAN POSITIONS FROM CENTERED PLATES, COMPARISON STARS *k*, *p*, *a*

Instr.	No. Pl.	*i*	*b*	*d*	*f*	*o*	*n*	*m*	*e*	*h*	*q*	*j*	Mean ±
						Right Ascension, Unit = 0ˢ001							
100-in. Cass.	9	0	−1	0	0	+1	+1	0	−1	0	−2	+1	0ˢ0008
60-in. Cass.	4	0	+1	0	+1	−1	−2	0	+1	+1	+2	−2	.0008
100-in. Newt.	4	−1	−2	0	−1	+1	+1	+1	−3	0	0	+1	.0010
60-in. Newt.	5	+3	+7	+6	+2	+4	+3	1	+2	1	0	0	.0026
60-in. Z.C.	5	+2	−5	0	+1	+1	0	4	+2	3	−3	+6	.0024
40-in. Refr.	6	0	+3	−1	...	−3	...	...	...	...	...	...	.0017
24-in. Refl.	4	+1	−17	−17	+5	+9	+30	22	−8	5	+8	+18	.0125
24-in. Z.C.	3	+6	−7	+13	+4	+24	+14	+7	+2	+14	+12	+6	.0100
10-in. Bruce	3	−15	+28	+33	−6	0	−22	−23	...	...	...	...	0.018
						Declination, Unit = 0″01							
100-in. Cass.	9	...	...	...	...	...	...	...	...	...	...	...	...
60-in. Cass.	4	+2	+6	+1	+1	+1	−5	−5	+1	−8	−4	−6	0″036
100-in. Newt.	4	+2	+5	+3	0	+2	−2	−1	0	0	−1	+1	.015
60-in. Newt.	5	−8	0	−1	2	+4	+6	+10	3	+12	+11	+11	.062
60-in. Z.C.	5	+6	+9	0	1	+1	−2	−2	7	+5	+9	−2	.040
40-in. Refr.	6	+6	+5	+5	...	−1	...	...	...	...	...	...	.042
24-in. Refl.	4	−44	−19	+4	68	+16	+13	−13	34	+20	+15	+9	.230
24-in. Z.C.	3	+7	+28	+53	+84	+16	+13	+19	+25	+34	+5	+47	.300
10-in. Bruce	3	−6	+34	+53	+46	+69	+1	−19	...	...	...	...	0.33

gives the mean differences $0^{s}.0027$ and $0''.026$ for bright comparison stars and $0^{s}.0007$ and $0''.015$, respectively, for faint comparison stars. This agreement between the two forms of the 100-inch telescope is remarkable and speaks well for the accuracy possible with the Newtonian combination for stars limited to about $6'$ from the center of the field, even when the range in magnitude is considerable. The importance of keeping near the axis will be seen later on. Comparison of the results for the faint star q, with a large dependence factor on the bright comparison star A, shows that at $6'$ from the axis the error of the 100-inch Newtonian is inappreciable in spite of the great difference in magnitude.

The results for the centered plates in Tables IV do not indicate much choice between the 60-inch Newtonian with and without the corrector lens. The relative merits of these combinations will receive consideration in the study of Tables V and VI.

Comparing in a similar manner the 40-inch Yerkes refractor with the Mount Wilson 100-inch Cassegrain, we find the mean difference, for bright comparison stars, in α, $0^{s}.0033$, in δ, $0''.083$. With faint comparison stars these differences are $0^{s}.0022$ and $0''.042$, respectively, with reduced weight, however. The right-ascension differences are larger than those found in the comparison of the two Mount Wilson Cassegrain telescopes and the 100-inch Newtonian. The differences in declination are abnormal and difficult to explain (see discussion of atmospheric dispersion, p. 264).

In view of the higher aperture ratio of the Yerkes 24-inch reflector, the average errors shown in Tables IV are not unduly large. They are not materially diminished when the corrector lens is used.

The results for the 10- and 3-inch lenses show that positions obtained with these instruments are sufficiently accurate for many purposes. In the case of the 3-inch Ross, the average errors, reduced to linear measure, are, in α and δ respectively, 1.2 and 1.6 microns, which is all that can be expected of a fast, coarse-grained emulsion.

Tables V, giving the dispersion for each telescope and each star, include results of great interest, applicable, as already remarked, to the important problems of parallax and proper motion. Between the 100-inch Cassegrain and the 60-inch Cassegrain there does not seem much to choose. The 100-inch Newtonian shows up nearly as

TABLE V*a*

AVERAGE RESIDUALS FROM CENTERED PLATES, COMPARISON STARS A, B, C

Instr.	No. Pl.	*g*	*c*	*k*	*i*	*b*	*d*	*p*	*a*	*f*	*o*	*n*	*m*	*e*	*l*	*h*	*q*	*j*	Mean
									it Ascension, Unit = $0^s.0001$										
100-in. Cass.	9	14	7	13	19	16	12	13	20	21	29	28	13	18	22	24	21	28	20
60-in. Cass.	4	47	10	12	18	32	25	18	25	18	27	22	15	37	42	28	18	25	24
100-in. Newt.	4	42	3	18	37	28	18	30	22	20	13	50	20	30	28	18	30	18	25
60-in. Newt.	5	28	42	40	50	56	52	86	68	70	86	74	58	88	64	84	108	54	65
60-in. Z.C.	4	22	52	38	25	25	57	42	57	47	30	20	42	55	20	45	25	40	38
40-in. Refr.	6	28	10	43	33	33	35	60	37	.....	30	.....	.....	.....	.....	.....	.....	.....	34
24-in. Refl.	4	125	117	357	217	205	242	265	58	317	217	157	45	130	55	110	172	220	177
24-in. Z.C.	3	63	73	37	50	70	83	87	97	33	50	37	83	97	27	100	110	110	71
									Declination, Unit = $0''.001$										
100-in. Cass.	9	27	23	52	34	44	34	27	48	38	40	31	44	43	57	34	25	34	37
60-in. Cass.	4	42	28	52	38	38	20	60	40	52	62	60	42	92	88	50	28	78	51
100-in. Newt.	4	102	67	30	50	22	52	40	50	15	32	38	75	50	95	38	65	40	50
60-in. Newt.	5	42	25	40	18	35	80	87	35	28	70	25	57	45	32	55	102	0	46
60-in. Z.C.	4	42	32	70	28	60	48	68	48	50	54	48	52	44	38	66	42	46	49
40-in. Refr.	6	12	13	40	27	22	57	53	42	.....	77	.....	.....	.....	.....	.....	.....	.....	38
24-in. Refl.	4	112	87	220	195	170	218	152	454	260	265	218	372	240	150	242	285	162	224
24-in. Z.C.	2	120	80	0	90	40	120	80	160	130	40	210	160	80	80	40	120	90	97

TABLE V*b*

AVERAGE RESIDUALS FROM CENTERED PLATES, COMPARISON STARS *k*, *p*, *a*

Instr.	No. Pl.	*i*	*b*	*d*	*f*	*o*	*n*	*m*	*e*	*h*	*q*	*j*	Mean
	Right Ascension, Unit = 0ˢ.0001												
100-in. Cass.	9	18	13	16	17	19	22	17	18	24	14	20	18
60-in. Cass.	4	10	22	25	20	15	20	15	38	15	15	10	19
100-in. Newt.	4	23	22	8	20	22	37	15	25	22	28	22	22
60-in. Newt.	5	26	52	46	46	30	20	26	80	60	52	28	42
60-in. Z.C.	4	22	54	40	34	22	10	30	56	20	36	10	30
40-in. Refr.	6	20	55	32	......	43	......	......	......	......	......	......	37
24-in. Refl.	4	117	102	102	130	132	102	147	57	57	85	130	105
24-in. Z.C.	3	0	123	37	123	23	110	160	50	27	133	173	87
	Declination, Unit = 0″.001												
100-in. Cass.	9	18	27	28	28	22	28	31	29	29	25	20	26
60-in. Cass.	4	38	25	18	45	18	25	38	85	8	50	42	36
100-in. Newt.	4	23	15	15	12	20	8	62	35	18	68	32	28
60-in. Newt.	5	36	86	84	42	60	92	54	54	56	48	30	58
60-in. Z.C.	4	46	18	22	64	86	82	26	86	34	72	70	55
40-in. Refr.	6	18	37	60	......	62	......	......	......	......	......	......	44
24-in. Refl.	4	220	392	175	272	172	155	207	285	225	265	285	241
24-in. Z.C.	2	0	80	0	40	0	80	290	160	170	250	70	104

TABLE VI*a*

MEAN POSITIONS FROM DECENTERED PLATES, COMPARISON STARS A, B, C

Instr.	Displ.	No. Pl.	*g*	*c*	*k*	*i*	*b*	*d*	*p*	*a*	*f*	*o*	*n*	*m*	*e*	*l*	*h*	*q*	*j*	Mean ±
									Right Ascension, Unit = 0ˢ001											
60-in. Cass. . .	10′ W.	2	− 1	− 4	+ 1	− 2	− 4	− 7	+ 1	− 7	− 3	+ 2	0	− 2	− 2	− 5	− 1	+ 2	− 7	0ˢ0030
100-in. Newt. . .	8 E.	4	− 1	+ 3	+ 8	+ 7	+ 9	+ 12	+ 29	+ 5	+ 19	+ 35	+ 39	+ 33	+ 19	+ 38	+ 21	. . . . .	+ 32	.0194
60-in. Newt. . .	10 E.	2	+ 1	+ 8	+ 12	+ 6	+ 6	+ 15	+ 40	+ 8	+ 32	+ 35	+ 51	+ 48	+ 24	+ 39	+ 26	+ 47	+ 35	.0255
60-in. Newt. . .	10 W.	2	+ 3	− 1	− 2	− 15	− 25	− 22	− 20	− 9	− 38	− 26	− 32	− 41	− 32	− 37	− 34	− 40	− 34	.0242
60-in. Z.C. . . .	10 E.	3	− 3	− 6	− 3	+ 4	− 12	− 6	+ 3	− 9	− 7	− 6	+ 4	+ 9	− 9	+ 12	0	− 2	+ 5	.0059
40-in. Refr. . . .	12 E.	6	+ 5	− 1	− 10	− 6	− 3	− 5	− 20	− 4	. . . . .	− 18	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	.0080
24-in. Refl. . . .	12 E.	2	+ 7	+15	+ 4	+ 33	+ 34	+ 62	+116	+ 79	+ 88	+133	+134	+130	+109	+116	+ 86	+145	+135	.084
24-in. Refl. . . .	27 E.	1	−13	+59	+279	+228	+214	+217	+252	+224	+268	+238	+294	+290	+229	+231	+271	+200	+305	.224
24-in. Z.C. . . .	12 E.	3	− 5	− 6	− 4	+ 5	+ 10	+ 16	+ 2	+ 2	+ 10	+ 17	+ 9	+ 2	+ 16	+ 19	+ 14	− 1	+ 34	0.010
									Declination, Unit = 0″01											
60-in. Cass. . .	10′ W.	2	0	+ 1	+ 7	+ 6	+ 9	+ 5	+ 5	+ 9	+ 4	+ 5	0	− 2	+ 3	+ 5	− 2	− 4	+ 1	0″040
100-in. Newt. . .	8 E.	4	+ 2	− 3	0	− 5	− 2	− 1	− 3	− 1	− 5	+ 4	+ 2	+ 4	− 4	+ 14	− 1	. . . . .	2	.033
60-in. Newt. . .	10 E.	2	+ 8	+ 1	− 9	− 6	− 9	− 5	+ 3	+ 8	− 6	+ 11	+ 9	+ 21	− 3	+ 35	+ 13	+ 21	10	.105
60-in. Newt. . .	10 W.	2	+ 7	− 1	− 26	+ 5	+ 2	− 7	− 14	+ 28	− 6	0	− 1	+ 8	− 5	+ 36	+ 6	+ 1	5	.093
60-in. Z.C. . . .	10 E.	2	+ 8	− 1	+ 3	− 5	− 6	− 7	+ 9	− 8	+ 5	+ 12	− 5	− 2	− 3	− 6	+ 1	+ 4	1	.050
40-in. Refr. . . .	12 E.	6	+ 3	+ 3	− 7	− 10	− 7	− 11	− 17	− 14	. . . . .	− 17	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .	.099
24-in. Refl. . . .	*	3	− 8	−10	+ 45	+ 36	+ 57	+ 56	+ 91	+ 70	+ 69	+ 89	+ 99	+118	+ 77	+142	+ 83	+125	82	.74
24-in. Z.C. . . .	*	2	+ 5	+ 4	+ 25	+ 18	+ 27	+ 56	+ 35	− 6	+104	+ 35	+ 43	+ 32	+ 40	+ 14	+ 47	+ 35	+ 62	0.35

* Displaced in α and δ.

TABLE VI*b*

MEAN POSITIONS FROM DECENTERED PLATES, COMPARISON STARS, *k*, *p*, *a*

Instr.	Displ.	No. Pl.	*i*	*b*	*d*	*f*	*o*	*n*	*m*	*e*	*h*	*q*	*j*	Mean ±
							Right Ascension, Unit = $0^s.001$							
60-in. Cass.	10′ W.	2	+ 1	+ 1	− 2	− 1	− 2	0	− 1	+ 1	+ 3	+ 1	− 3	$0^s.0015$
100-in. Newt.	8 E.	4	− 4	+ 5	+ 4	+14	+ 11	+ 9	+ 2	+12	9	....	+11	.0081
60-in. Newt.	10 E.	2	− 7	0	+ 6	+25	+ 2	+11	+ 5	+15	7	− 1	+ 6	.0077
60-in. Newt.	10 W.	2	+ 5	− 4	+ 3	−21	− 7	− 9	− 7	− 9	1	− 14	− 6	.0078
60-in. Z.C.	10 E.	3	+ 8	+16	0	− 3	− 5	+ 4	+ 8	− 1	6	− 3	+ 8	.0056
40-in. Refr.	12 E.	6	0	+ 2	0	....	− 1	....	....	....	....	....	....	.0008
24-in. Refl.	12 E.	2	−16	−11	−28	+59	+ 26	+ 2	−19	+62	23	+ 15	+26	.026
24-in. Refl.	27 E.	1	−16	−32	−28	+18	− 15	+63	+68	−23	+35	− 35	+69	.036
24-in. Z.C.	12 E.	3	+ 4	+30	+21	+ 7	+ 14	+ 6	− 6	+17	0	+ 8	+34	0.013
							Declination, Unit = $0''.01$							
60-in. Cass.	10′ W.	2	0	+ 2	− 1	− 3	0	− 5	− 8	− 3	+ 6	− 9	− 6	$0''.039$
100-in. Newt.	8 E.	4	0	+ 1	+ 1	− 7	+ 3	+ 3	+ 8	− 5	+ 2	....	+ 1	.028
60-in. Newt.	10 E.	2	− 7	− 8	− 6	− 3	− 5	+ 1	+11	− 3	+12	0	+ 2	.053
60-in. Newt.	10 W.	2	+ 1	− 4	− 9	0	+ 10	+ 3	− 1	−10	− 3	+ 3	− 3	.043
60-in. Z.C.	10 E.	2	0	+ 3	− 1	+ 6	+ 7	−11	− 5	+ 2	+ 4	− 5	− 5	.044
40-in. Refr.	12 E.	6	+ 3	+ 2	0	....	− 2	....	....	....	....	....	....	.018
24-in. Refl.	12 E.	2	−32	− 5	+ 9	+15	− 3	+ 7	− 1	+15	+ 5	+ 19	+ 6	.106
24-in. Refl.	27 E.	1	+ 8	−49	+26	+18	+116	+91	+55	−14	+69	+111	+73	.570
24-in. Z.C.	12 E.	3	+ 7	+28	+53	+84	+ 16	+14	+19	+25	+34	+ 5	+47	0.300

well. The Yerkes 40-inch refractor falls somewhat behind. The corresponding probable error computed for the 100-inch Cassegrain, faint comparison stars, for which the average residual is $0^{s}.0018$, is $0^{s}.0015$, or $0''.022$, for a single plate. This value is smaller than usually obtained in parallax determinations, which have the advantage of a larger number of comparison stars, five as compared with three used here, and the additional advantages of a careful equalization of magnitudes and all-round symmetry in the choice of comparison stars. Even with the very bright comparison stars, whose images on the plate are as much as $4''.0$ in diameter, the accuracy is nearly as great. It is important to note that the dispersion does not increase with decreasing brightness of the stars, as one might expect—additional evidence that there is little to fear in obtaining the positions of faint stars from much brighter stars used as standards.

It is to be noted that in right ascension the results with the correcting lens are considerably better than without, especially in the case of the bright comparison stars. In declination no improvement is shown. I believe this circumstance to be due to astigmatism of the mirror, as it has been found very difficult to obtain good images with the correcting lens when the form of the mirror is below normal, as, for example, in the early evening. In the case of the correcting lens used with the 24-inch Yerkes reflector, considerable improvement has been obtained.

8. DECENTERED PLATES, TABLES VI

100-inch Cassegrain.—Since it was not possible without some readjustments to use a plate larger than 5×7 inches, only exposures with field central could be made with this instrument. The residuals for star q (Table IVa) give, however, no indication of a field effect. This star lies close to comparison star A (Fig. 1), which is $5'.5$ from the axis, and its residuals (Table Va) are even less than the average for all of the stars, in spite of the great difference in magnitude. This of course does not prove that there is no constant error in the position of q, but any such error must be small, since the mean positions of q from the three instruments, 100-inch Cassegrain, 100-inch Newtonian, and 60-inch Cassegrain, agree very well (Table IVa).

A constant error is not of so much importance, however, as the size of the dispersion in Tables V.

60-inch Cassegrain.—Two plates were taken by van Maanen with star *g* displaced 10′ W. Any field-magnitude effect should appear in the positions of stars *a*, *b*, and *d*, since these are located near star C, at a distance of 12′ from the axis, and average 3.3 mag. fainter than C. The average error in the mean position of these stars (Table VI*a*) is $0^s.0060$ in α and $0''.077$ in δ, as compared with averages of $0^s.0024$ and $0''.032$, respectively, for the remaining stars. It may be concluded that a small field effect occurs when the range or difference in magnitude and in field angle is large. The residuals obtained with the fainter system of comparison stars (Table VI*b*) are remarkably small and of the same order as for centered plates. Except for the very brightest stars, therefore, no appreciable field-magnitude effect occurs with this instrument.

100-inch Newtonian.—Four plates were obtained by van Maanen with the field displaced 8′ E. The effect of coma is strikingly shown in the right-ascension residuals (Table VI*a*), the average being $0^s.0194$ as compared with $0^s.0030$ for field central, or more than six times greater (Table IV*a*). The declinations show little effect, as was to be expected. For several stars the error in the mean right ascension amounts to nearly $0''.6$. The largest individual error, $1''.05$, occurred for star *q*. The advantage of fainter comparison stars is well shown in Table VI*b*, where the average error in α is reduced to $0^s.0081$, or less than half. The largest individual error is reduced to $0''.4$.

60-inch Newtonian.—Many plates were rejected on account of astigmatism of the mirror. Table VI*a* shows the average errors for two pairs of plates with the field displaced 10′ E. and 10′ W., the mean for the four plates being $0^s.0238$, or slightly greater than for the 100-inch Newtonian, for which the displacement was only 8′. The behavior of the two instruments is therefore closely parallel. Comparison of the plates displaced east and west clearly shows a reversal of the sign of the error, a circumstance of course to be expected. Reduced with the fainter comparison stars, the average error is $0^s.0077$, or only one-third. In declination, the average error with bright comparison stars is $0''.099$; with faint stars, $0''.048$. Comparison with the corresponding values for the 100-inch Newtonian,

0".033 and 0".028, respectively, shows the great superiority of the results in declination obtained with the latter instrument, undoubtedly owing to the better figure of the mirror. The longer focal length of the 100-inch is, however, of no advantage for the displaced field, as shown by the right ascensions.

60-inch Newtonian with zero-power corrector.—Table VI*a* shows that the average errors with the correcting lens are $0^s.0059$ and 0".050, as compared with $0^s.0248$ and 0".099 without. With fainter comparison stars the improvement is not so marked, $0^s.0056$ and 0".044 as compared with $0^s.0077$ and 0".048. There is no doubt that the performance of the corrector lens can be considerably improved by removing the astigmatism of the mirror, which, by combining with a residual spherical aberration of the corrector lens, seriously affects the form of the images. Plates taken in the early evening when the astigmatism of the mirror is a maximum show this effect well.

Yerkes 40-inch refractor.—Six plates were centered 12′ E. For the bright comparison stars the average errors are large, $0^s.0080$ and 0".099. The notably small residuals for faint comparison stars unfortunately are not of high weight since they depend on only four stars. Tables IV and VI show that the Yerkes 40-inch refractor is nearly on a par with the two Mount Wilson Cassegrain telescopes in the matter of mean positions when the fainter system of comparison stars is used, but falls behind when the comparison stars are bright. From Tables V, it appears also to fall behind in the dispersion of the individual values from the centered plates. The large residuals, 0".3 for stars *p* and *o* in Table VI*a*, are undoubtedly a field-magnitude effect and agree in sign with the known inward coma of the 40-inch lens.[10]

Yerkes 24-inch reflector.—Residuals are given for two plates centered 12′ E. and for one 27′ E. The abnormal distance was included since the positions of asteroids are sometimes measured at this distance from the axis. The average error in right ascension for the first group is three times that of the Mount Wilson 60-inch Newtonian, the ratio of their focal lengths (Table VI*a*). It must be remem-

[10] F. E. Ross, *Astrophysical Journal*, **76**, 190, 1932.

bered that the high speed-ratio of the 24-inch is an unfavorable factor in precision measurements. For the faint comparison stars the accuracy is considerably greater, the residuals being reduced to one-third for 12′ E. and to one-sixth for 27′ E. The largest individual errors in right ascension on one plate are 2″.9 for 12′ E. and 4″.6 for 27′ E.; for the faint comparison stars they are 1″.05 and 1″.07, respectively. It is difficult to understand the large errors in declination, which are seven times those of the 60-inch Newtonian with bright comparison stars. With the faint comparison system, the difference is normal.

Yerkes 24-inch reflector with correcting lens.—Table VI*a* shows considerable gain in accuracy, eightfold that without the corrector lens for bright comparison stars, and twofold that for the faint comparison stars. The declinations, however, are quite unsatisfactory. A strongly elliptical field was obtained with this corrector, which persisted even on rotation. The mirror itself, however, was not rotated. The lens was sent to Mr. Fecker for changes designed to improve the field.

9. ATMOSPHERIC DISPERSION

Atmospheric dispersion is a factor to be considered in precise astrometry. In the present series of measures it affects the right ascensions only slightly, since all plates were exposed near the meridian. The effect is twofold: (*a*) for all stars, the images—in reality spectra—should be of tadpole shape with a reflecting telescope or of a comatic flaring type in the case of refractors, the violet end shading off very gradually as compared with the blue end, so that a magnitude equation should appear; (*b*) if stars are of different colors, the distribution of light within each spectral image will vary with the color, thus giving rise to an equation or correction for stars of the same magnitude. In the case of exposures with the Yerkes 40-inch refractor, in which yellow, or yellow and red-sensitive, plates are used in conjunction with a yellow filter, a similar effect is present, the form and distribution of light depending on the color correction of the objective. It is of interest to calculate the amount of this effect for the Mount Wilson and the Yerkes plates for the field photographed, assumed to be on the meridian.

Mount Wilson reflector plates.—Correcting for the mean height of the barometer and using values for the atmospheric dispersion given in the *Smithsonian Physical Tables* (6th ed.), page 193, assuming that blue-sensitive plates are used, and assuming further a range of sensitivity from F to K, we find the length of the spectrum to be $0''.33$. For the brighter stars the length should be even greater. It is therefore reasonable to assume that the displacement of a faint star relative to a bright one is of the order of $0''.10$.

Yerkes refractor plates.—Exposures were made on Wratten and Wainwright plates, with a high sensitivity in the red. On the assumption that the sensitivity extends from C to F and that a yellow filter is used, the length of spectrum becomes $0''.43$. If isochromatic plates are used, sensitivity ranging from D to F, the length of spectrum is reduced to $0''.31$. Since the spectrum is more sharply cut than in the case of the reflector plates, which are subject to the effect of troublesome ultra-violet light, the additional length of spectrum in the case of the panchromatic plates is, in a comparison of the two types of telescopes, neutralized.

10. ON THE ACCURACY OF MEASUREMENT OF LARGE IMAGES

It has been shown in the case of the Cassegrain telescopes of the Mount Wilson Observatory that little is to be feared in obtaining precise positions of faint stars relative to neighboring bright ones for a total range of at least 6 mag. and for field angles as great as $10'$. The images of the brighter stars measured with the 100-inch Cassegrain were as large as 0.90 mm ($4''.5$) in diameter, the faintest as small as 0.15 mm ($0''.75$). The accuracy reached means that these large images must have been bisected with an error of less than 1 per cent. It is important that this estimate be independently confirmed. One of the 100-inch Cassegrain plates with two exposures to the standard field was selected for a special test and the two images of star *g*, which were the largest, were measured. It was not discovered until later that this star is a double (Burnham 464, $8^{m}.5$ and $9^{m}.5$, $s = 1''.20$, $p = 111°$), since, owing to its brightness, the images are nearly round on almost all the plates. For our purpose, however, duplicity is an advantage, as the images become irregular in outline, thus making the test more severe. One (*B*) was even slightly elongated.

The test was as follows: The position of each image of g (A and B) was measured with respect to a very fine bright spot S near by, a procedure which throws all the error into the bisections of the star images and allows the left-right discordance to be obtained in its full amount, and not differentially as is usually the case. Ten observers took part in the measurement, all except one experienced in measuring. Each observer made three sets of measures, a set consisting of two measures, plate direct and reversed, respectively. The results are given in Table VII, arranged in order of increasing $L-R$ discordance. The values of mean Δ in parentheses are disregarded

TABLE VII

PERSONALITY IN BISECTION

Observer	Left-Right			Mean Δ		Bisection Point
	SA	*SB*	Mean	*SA*	*SB*	
	mm	mm	mm	mm	mm	mm
A	−0.031	−0.042	−0.036	8.872	7.609	0.480
B	− .013	− .019	− .016	.870	.612	.503
C	− .017	.000	− .007	.873	.607	.475
D	+ .002	+ .008	+ .005	.872	.606	.498
E	+ .013	+ .008	+ .010	.871	.603	....
F	+ .027	+ .024	+ .025	.870	.615	.493
G	+ .016	+ .048	+ .032	.865	.609	.477
H	+ .049	+ .047	+ .048	.866	.600	.493
I	+ .044	+ .055	+ .049	.868	.605	.493
K	+0.139	+0.143	+0.141	(8.859)	(7.587)	0.451
Mean	....	....	....	8.870	7.607	....

on account of the abnormally large values of $L-R$ for observer K. The columns "Mean Δ," observers A–I, show a doubtful correlation with $L-R$ discordance, which, however, is strengthened if results for observer K are included.

From the values of mean Δ it is seen that the error of the average observer is 0.0019 mm for image A and 0.0034 mm for B. Since image B is very irregular and difficult to measure, we may take 0.002 mm as the average deviation from the mean observer. It is difficult to believe that such accuracy in the measurement of images of this size is possible, even without the wide penumbra which actually surrounds these large images. The question arises, What is the

criterion, conscious or unconscious, of a bisection? It is evident from the foregoing results that it is not a matter of space perception, since the wide penumbra would preclude the accuracy actually attained, which is one four-hundred-and-fiftieth part of the diameter of the image. The *L–R* differences furnish a clue, provided it can be shown that there is no correlation between a spatial-bisection personality and the *L–R* personality. In order to test the correlation, all the observers but one participated in a further series of experiments with the same measuring instrument. Each observer set the thread upon two fine spots close together on a photographic negative and upon what he considered to be the mid-point between them. Three runs were made by each observer. The angle subtended at the eye by the spots was 9°. All observers agreed upon the separation of the spots, 0.595 mm, the maximum difference being 0.001 mm, but the readings for the mid-point differed widely, as shown by the last column of Table VII. Except for observer K, there is no correlation between bisection personality, which is a spatial perception, and *L–R* personality. Observer K, however, sets the wire too far to the left in both cases, his *L–R* discordance corresponding to a bisection point of 0.42. It seems probable that the cause of the large personality in the case of observer K is spatial perception. On the other hand, observer A in bisecting an image sets the wire too far to the right, and in space bisection, too far to the left.

Since the bisection of an image as a spatial perception seems thus to be ruled out, an attractive alternative hypothesis is that the bisection of an image is in reality and unconsciously an estimate of magnitude, the observer, in the process of making a bisection, halting the thread when he has divided the image into two apparently equal magnitudes, barring the exceptional cases of badly distorted images. This hypothesis is attractive in that it makes the *L–R* discordance in bisection and the well-known *L–R* discordance in the comparison of magnitudes in photometry depend upon the same physiological-optical phenomenon, whatever that may be. I have no doubt that the connection between the two phenomena has already been noted by others. Any theory proposed must explain the very large personality in the case of observer K, who is experienced and consistent in his settings. It may be remarked incidentally that the mo-

tion of the measuring thread has nothing to do with the personality, for comparisons have been made with the wire at rest. In carrying the subject further it would be of great interest to determine the effect of a change in the relative direction of measuring thread and line joining the observer's eyes.

11. PERSONALITY AND MAGNIFICATION IN PLATE MEASUREMENT

All plates except those taken at the 100-inch Cassegrain were measured with a magnification of 40 at the comparator. This value is unusually high, but was found to give less eyestrain, especially as a broad measuring wire was used, which added to the comfort and ease of measurement. For the 100-inch Cassegrain plates the magnification was 13. It is important to determine if the measurement depends on the magnification. Theoretically, one would expect some such effect if the images are at all irregular, with differences in the penumbral shading in the direction measured. Since the lower power would tend to incorporate some of the penumbra into the image itself, the apparent size of the image would vary asymmetrically with magnification. To test this, a decentered exposure made with the 60-inch Newtonian was selected for special measurement under magnifications of 13 and 40, respectively. On account of the strong coma in a decentered plate, which causes an unsymmetrical penumbra, the effect, if present, should be pronounced. Measurement was in the usual way, but in right ascension only, for which the effect should be a maximum, and was limited to the brighter stars *g* and *c* relative to comparison stars A, B, and C. The diameter of image *c* is 0.33 mm; of the others, 0.40 mm. Three observers took part in

TABLE VIII

MAGNIFICATION PERSONALITY

Observer	*g* (α)			*c* (α)		
	Mag. 40	Mag. 13	Diff.	Mag. 40	Mag. 13	Diff.
G.	^{s}0.690	^{s}0.690	0^{s}000	^{s}0.148	^{s}0.145	+0^{s}003
G (repeat) . . .	.685	.686	− .001	.143	.139	.004
B.	.686	.683	+ .003	.137	.132	.005
B (repeat) . . .	.686	.686	000	.137	.134	.003
L.	.683	.681	+0.002	.137	.134	+0.003

the measurement, all of great experience. Two of the observers repeated their measures after the lapse of some days.

Table VIII shows that under the extreme conditions chosen—namely, large images affected by considerable coma—there is a difference in the results for star *c* but not for *g*. The agreement of all three observers in the matter of the difference depending on magnification is remarkably good. Aside from this, a marked personality is noted, the maximum being $0\overset{s}{.}010$, corresponding to 0.006 mm on the plate, or about 1.5 per cent of the diameter of the images measured.

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ON THE SPECTRUM OF HYDROGEN SULPHIDE

By NORMAN O. STEIN

ABSTRACT

The emission spectrum of H_2S was investigated by means of a valve oscillator, the circuit of which is described in detail. *No spectrum characteristic* of H_2S was observed, the gas giving the Balmer series, the hydrogen secondary spectrum and the continuous spectrum, along with a group of diffuse bands extending from about 2470 A to 2860 A.

Carbon disulphide shows the same bands when excited under similar conditions and they are therefore ascribed to sulphur.

The wave-lengths of these bands are given in a table.

The purpose of the work outlined below was to observe, if possible, a characteristic emission spectrum of hydrogen sulphide. On analogy with the *OH* bands obtained in the case of water vapor it was deemed likely that under the conditions described below *SH* bands might be observed. W. H. Bair[1] has described attempts made to excite a characteristic spectrum of H_2S, in which he used a discharge tube containing internal electrodes and excited by means of the transformer discharge using fairly high potentials. It was decided that a discharge employing lower energy than that which is normally used in discharge technique would be more favorable to the production of *HS* bands. Such an energy system is provided by a valve-oscillator circuit.

On the suggestion of Mr. S. V. R. Rao of this laboratory, who has had considerable experience with this type of discharge, an oscillating circuit, described below, was used.

Although the experiments did not yield a characteristic spectrum of H_2S, it was thought that a description of the method and the band spectrum obtained would prove of interest.

APPARATUS AND METHOD

The valve-oscillator circuit is shown in Figure 1. A coil *A*, of 1400 micro-henrys, was connected between the grid and the negative filament terminal of a Marconi LS5-power valve. Between the anode of the valve and the high-tension positive, a coil *B* of 9.5 micro-henrys was connected which could be coupled to the coil *A*. Across

[1] *Astrophysical Journal*, **52**, 301, 1920.

the coil *B* a variable condenser of 0.001 M.F. was connected, in order to control the oscillations. The discharge tube was connected across a third coil, *C*, of 200 micro-henrys, which was coupled to the coil *B*. The 220-volt, D.C. mains were used to supply the anode potential and a reservoir condenser of 0.5 M.F. was connected across the mains. The filament current was supplied by a 6-volt accumulator and was controlled by means of a variable resistance with a voltmeter in the circuit. The current produced in the discharge tube was of the order of a few milliamperes and the voltage was less than 200. The frequency was about 7,000,000 cycles.

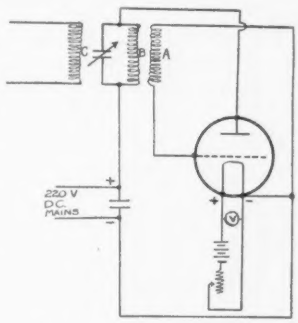

FIG. 1.—Valve-oscillator circuit. The circuit is described in text.

The discharge tube was of soft glass, 6 inches long and 1 inch in diameter. One end was sealed and the other was flanged and ground plane, in order to affix a thin quartz window by means of a thin layer of De Khotinsky cement. Two tubes were sealed in at the side to conduct the gas into the tube. The electrodes were external and consisted of two rings of aluminum foil which were placed about 1 inch apart. The discharge tube was used in the end-on position. Spectrograms were taken with a Hilger E3 quartz instrument and panchromatic plates were used.

The H_2S was prepared by decomposing magnesium hydrosulphide which was produced by allowing H_2S from a Kipp to react with a suspension of finely divided magnesium oxide in water. The gas was thoroughly dried over P_2O_5 and condensed into the trap *A*, Figure 2, by means of liquid air. The whole apparatus was then pumped out with a high-vacuum oil pump and the liquid-air jacket replaced by one containing CO_2-alcohol. The apparatus was swept out several times with H_2S before the discharge was started. The pressure was controlled by taps and a large-volume bulb *B*. Spectrograms were taken with H_2S streaming between the traps *A* and *C*. By closing appropriate taps spectrograms could be made of the static gas. A fine slit was used and exposures of from one to eight hours were made with various conditions of discharge and pressure. Comparison spec-

trograms were made of pure hydrogen in a similar discharge tube placed alongside that used for the H_2S. Comparison spectra of the iron spark were juxtaposed on each spectrum taken and a wave-length scale was photographed alongside to facilitate the recognition of iron lines. Subsequently the spectrum of carbon disulphide was excited in a similar discharge tube under the same conditions. Mi-

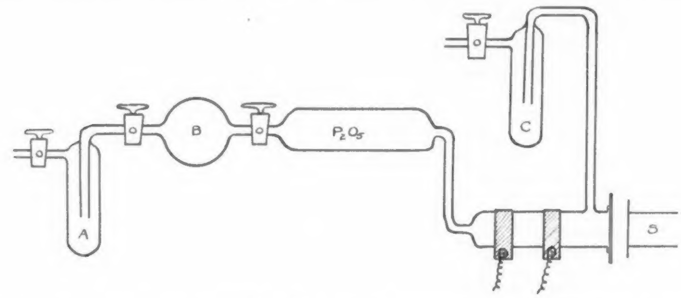

FIG. 2.—Apparatus for preparing H_2S for observation. The apparatus is described in text.

crophotometer photographs were taken of the parts of the spectrograms which showed characteristic bands.

EXPERIMENTAL RESULTS

A careful examination of the plates showed that the spectrum of H_2S differed from that of hydrogen in the appearance of a number of diffuse bands, which appeared to be degraded toward the red, and extended from about 2470 A to 2860 A. The H_2S spectrum always showed the Balmer series and the hydrogen secondary spectrum along with a background of the hydrogen continuous spectrum which extended to the limit of the instrument, i.e., about 2100 A. The hydrogen and H_2S plates were identical except for these bands. Spectrograms of carbon disulphide also showed these bands more strongly. They therefore cannot be regarded as characteristic of hydrogen sulphide. The wave-lengths of the strongest parts of these bands were determined as accurately as possible, considering their diffuse character and the difficulty of setting the micrometer accurately. The values are given in Table I. This table also includes a portion of Table IV of W. H. Bair's paper[2] which falls in the same wave-length region and which tabulates bands ascribed by Bair to SO_2. An ex-

[2] *Loc. cit.*

amination of the table indicates that some of the bands which appear in both the H_2S spectrum and in that due to CS_2 appear to correspond in wave-length to those described by Bair. This seems to indicate that at least some of the bands ascribed by Bair to SO_2 may

TABLE I

Hydrogen Sulphide	Sulphur Dioxide (Bair)
2474	2472.8
2479	......
2508	2497.1
2510	2517.1
2525	......
2542	2543.1
2551	......
2557	......
2578	......
2590	2590.3
2594	......
2608	......
2622	2624.0
2665	2665.5
2680	......
2696	2700.5
2711	......
2733	......
2745	2745.7
2757	......
2772	......
2780	2781.9
2804	2794.1
2856	2829.1
	2861
	2866

possibly be due to sulphur, which is the common constituent of the three compounds.

In conclusion, the author wishes to thank the Department of Scientific and Industrial Research for a Senior award during the tenure of which this research was carried out.

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ON THE COLORS OF DIFFUSE NEBULAE HAVING CONTINUOUS SPECTRA

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ABSTRACT

Theoretical results indicate that if a nebula emitting a continuous spectrum consists of free atoms or molecules or of particles smaller than $\frac{1}{4}\lambda$, the color of the nebula must be appreciably bluer than that of the exciting star. The difference, star-nebula, at λ 3900 and λ 5000 is by theory +1.1 mag. Spectrographic observations of the nebula in the Pleiades give +0.3 mag. The difference, star-nebula, at λ 4250 and λ 4750 (the effective wave-lengths of the Yerkes photo-electric photometer) is by theory +0.5 mag. Actual observations with a potassium cell and two light-filters give $+0.1 \pm 0.05$ mag. It is suggested that the nebula consists of a mixture of large and small particles, most of the reflected light coming from the larger particles, which scatter without relation to λ.

In accordance with the theoretical ideas of H. N. Russell[1] and the extensive observational results of E. Hubble,[2] it is now generally believed that the luminosity of diffuse galactic nebulae is caused by neighboring stars. Hubble has found that practically every diffuse nebula is associated with one or more stars to which it owes its luminosity, and that the inverse-square law holds for the distribution of light in the nebulae. The few discordant cases, where no central star is observed or where the luminosity of the nebula is too great in comparison with the luminosity of the star, may be accounted for by complete or partial obscuration of the star by dark interstellar clouds.[3] According to Hubble, stars of spectral class B0 or earlier are invariably associated with nebulae emitting bright-line spectra; stars of class B2 or later are usually associated with nebulae emitting continuous spectra with absorption lines, and stars of class B1 are frequently observed in nebulae emitting continuous spectra with bright lines. The Orion nebula is the best-known example of the group giving bright-line spectra. The nebula in the Pleiades, investigated chiefly by V. M. Slipher,[4] gives a spectrum "which is a true copy of that of the neighboring star Merope and of the other bright stars of the Pleiades."[5] Several other nebulae were observed by Sli-

[1] *The Observatory*, **44**, 72, 1921; *Proceedings of the National Academy of Sciences*, **8**, 117, 1922.

[2] *Astrophysical Journal*, **56**, 162 and 400, 1922.

[3] Russell, Dugan, and Stewart, *Astronomy*, **2**, 819 ff., 1927.

[4] *Lowell Observatory Bulletin*, No. 55, 1912.

[5] *Ibid.*

pher, Hubble, and others, which gave spectra resembling those of the neighboring stars, continuous spectra with absorption lines of hydrogen, helium, etc. Because of the faintness of the nebular light, the linear dispersion was small in all these observations, and the slit was probably relatively wide. In the case of the nebula in the Pleiades, Slipher exposed approximately twenty-one hours, and in the case of the nebula near ρ Ophiuchi, which also gives a continuous spectrum, the exposure was twenty hours.[6] It is clear that only the major features of the nebular spectrum have been observed, and that detailed studies of the widths and intensities of the nebular absorption lines are not possible with the present observational technique. It is generally believed that continuous nebular spectra are produced by the reflection of starlight from the particles of the nebula, although Hubble mentions the possibility that the "process may be one of absorption and reëmission," in analogy with the processes believed to exist in nebulae having bright-line spectra.

Other nebulae having "reflection spectra" are N.G.C. 2068,[7] N.G.C. 7023,[8] observed by Slipher, and a number of others observed by Hubble.[9]

If the reflection theory is adopted for nebulae with continuous spectra, it is of interest to investigate the laws of illumination and of scattering which the particles of the nebulae obey. In 1920 Seares and Hubble[10] discovered that forty-seven stars surrounded by nebulosity exhibited appreciable positive color excesses; the color indices of these stars were approximately half a magnitude redder than the average color indices corresponding to their spectral types. This effect was tentatively attributed by the authors[10] to selective scattering in the matter of the nebulae, although other explanations were also mentioned. Several nebulae, the central stars of which had positive color excesses, gave continuous spectra.

In this paper we shall investigate certain consequences of the reflection theory of nebulae having continuous spectra and shall

[6] *Ibid.*, No. 75, 1916.

[7] Slipher, *Publications of the Astronomical Society of the Pacific*, **31**, 212, 1919.

[8] Pease, *ibid.*, **27**, 240, 1915; Slipher, *ibid.*, **30**, 63, 1918.

[9] *Op. cit.*, **56**, 173, 1922.

[10] *Ibid.*, **52**, 8, 1920; *ibid.*, **56**, 414–416, 1922.

show how information may be obtained regarding the size of the particles of the nebula.

When a beam of light is made to pass through a diffuse medium consisting of very small particles, such as the molecules of a gas, the light is scattered and the transmitted beam is measured according to the usual exponential formula,[11]

$$I = Ee^{-c(\lambda)} ,$$

where

$$c(\lambda) = \frac{32}{3} \frac{\pi^3(m-1)^2 H}{N\lambda^4} + \sigma H = \beta\lambda^{-4} + \alpha .$$

The first term of this expression, in which m is the coefficient of refraction, λ is the wave-length, H is the length of path in the diffuse medium, and N is the number of particles per unit volume, represents the law of scattering by small particles derived by Lord Rayleigh. The second term, in which σ is the absorption coefficient, represents the effect of pure absorption which is independent of the wave-length. Since $(m-1)$ is proportional to N, β is also proportional to N.

According to Schuster,[12] Rayleigh's law holds also for small solid particles not exceeding in size one-quarter of the wave-length of the incident light. For large particles the absorption is unselective. For particles of intermediate size the coefficient of scattering, β, is proportional to intermediate powers of the wave-length, λ^{-3}, λ^{-2}, and λ^{-1}, but, as Schönberg has shown in an important monograph[13] which we shall use throughout this paper, the range in size of particles which scatter according to λ^{-3}, λ^{-2}, λ^{-1} is very small, so that in a uniform mixture of particles of all sizes these intermediate powers may be safely neglected. Only in special cases where there is reason to expect that the size of the particles reaches a pronounced maximum of frequency near the critical value would it be necessary to consider these terms.

[11] E. Schönberg, *Mitteilungen der Sternwarte Breslau*, **3**, 53, 1932. For a discussion of Rayleigh scattering see Jean Cabannes, *La diffusion moléculaire de la lumière* (Paris, 1929). An interesting application to astronomical phenomena has been made by Charles Fabry, *Journal de physique*, **7**, 89, 1917.

[12] Schuster and Nicholson, *An Introduction to the Theory of Optics* (3d ed., 1924), p. 320.

[13] *Loc. cit.*

It is probable that in a nebula, large and small particles are present. The question is: Do they scatter predominantly according to β or according to a? The large color excesses found by Seares and Hubble might suggest that the first is true. If the color excess is known for any given nebula, and if, furthermore, the length of the path is known, it is possible to compute β; if, furthermore, m is known, we are able to derive the constant N and consequently the density of the medium. How such a computation may be made in practice was shown by Schönberg[14] for a tentative case of an A0 star the color of which was supposed to be approximately that of an F0 star. For actual color excesses of B-type stars the computations were made by W. Gleissberg.[15]

The amount of reddening in the transmitted beam depends upon the value of β, and is therefore proportional to N and to H. Only in the case of large H is there any chance of observing an appreciable amount of reddening.

Since we are dealing, in the case of the nebulae, with scattering, the light which is absorbed from the transmitted beam must come to us as light from the nebula, and if the former is redder than the incident light, E, the latter must be bluer. In other words, if we liken the exciting star to the sun and the nebula to our own atmosphere, we should expect that the color of the nebula would be distinctly bluer than the light of the star.

Following Schönberg,[16] the intensity of the light scattered by very small particles, at right angles to the incident beam, is

$$G(\lambda)=\mu I(\lambda)=\mu e^{-c(\lambda)}E(\lambda) ,$$

where

$$\mu=\frac{\pi(m^2-1)^2}{2N\lambda^4}=\frac{3}{64}\frac{(m+1)^2}{\pi^2 H\lambda^4}\beta .$$

The incident beam is reddened as it passes through the diffuse medium because of the factor $e^{-c(\lambda)}$, consequently $G(\lambda)$ should be expected to be redder in the outer parts of the nebula than in the inner. If the nebula is supposed to form a spherical shell around the

[14] *Ibid.*, p. 62.

[15] *Astronomische Nachrichten*, **246**, 329, 1932.

[16] *Op. cit.*, p. 59.

star and the color excesses of Seares and Hubble are directly attributable to scattering in the nebula, the color index of the latter should change by an amount numerically equal to that of the color excess as we pass from the inner parts of the shell to the outer. It ought to be possible to test this effect observationally for some of the objects observed by Seares and Hubble.[17]

However, in most cases the effect of reddening of the incident beam is small (since it depends on β). This is not true for the effect of change of color in the scattering medium. Near the inner edge of the nebula, where $c=0$, we have

$$G(\lambda)=\text{Const } \lambda^{-4}E(\lambda) .$$

Consequently, if the intensity of the light of star and nebula are measured in two different wave-lengths, λ_1 and λ_2, we get

$$\frac{G(\lambda_1)}{E(\lambda_1)}=\left(\frac{\lambda_1}{\lambda_2}\right)^{-4}\frac{G(\lambda_2)}{E(\lambda_2)} .$$

Accordingly, if we determine the difference between the brightness of the nebula and of the star at λ_1 and λ_2, and put

$$m_{\text{nebula}}-m_{\text{star}}=\Delta ,$$

we obtain

$$\Delta_{\lambda_1}-\Delta_{\lambda_2}=10 \log \frac{\lambda_1}{\lambda_2} .$$

Thus for two convenient pairs of values of λ_1 and λ_2:

$$\Delta_{\lambda\, 5000}-\Delta_{\lambda\, 3900}=1.1 \text{ mag.} ,$$

$$\Delta_{\lambda\, 4750}-\Delta_{\lambda\, 4250}=0.5 \text{ mag.}$$

This effect is independent of the color (or spectral type) of the exciting star, as well as of the constant β. If N is small, the total luminosity of the nebula is small, but its color at the inner edge is not affected; roughly speaking, it should be as much bluer than the star as the sky is bluer than the sun.

The numerical values obtained above, which refer to a nebula

[17] Conditions are much more complicated in the nebulae having emission spectra, and such a test would not apply to them.

consisting of only small particles, are so large that an observational test is not difficult, provided the nebula is bright enough to be observed in two separate wave-lengths.

We have chosen for this test the nebula in the Pleiades. A slit spectrogram was obtained with a quartz spectrograph attached to the 24-inch reflector of the Yerkes Observatory. The exposure time was seven hours. The linear dispersion of the plate is approximately 1000 A/mm. Careful tests were made proving that the spectrum was actually that of the nebula, and that scattered light in the atmosphere and in the instrument was not appreciable. On the same plate with the spectrum of the nebulosity surrounding Maia the star's spectrum was photographed for comparison. Accurate measurements were impossible on a plate of such small scale. However, we have made an approximate determination of the relative intensities of the two spectra at different wave-lengths on microphotometer tracings calibrated roughly with the aid of sensitometer exposures developed under similar conditions. The spectrum of the star seems to be slightly fainter in the violet than that of the nebula, and the most probable value from our tracings is

$$\Delta_{\lambda\,5000} - \Delta_{\lambda\,3900} = +0.3 \text{ mag.}$$

The uncertainty of this value is estimated to be about ± 0.2 mag. It is certain, however, that the observed value is much smaller than that obtained from theory, and it is very probable that there is a slight effect in the required direction.

In order to confirm our spectro-photometric observation, we have measured the color of the nebulosity surrounding the Pleiades with the photo-electric photometer attached to the 40-inch telescope. The filters used are designated Y_1 and B_1 and have effective wave-lengths of 4750 and 4250 A, respectively, when combined with the spectral sensitivity of the photo-cell. This combination of filters was chosen so that the photo-electric currents produced by a star of early spectral type would be the same through both filters. The difference in color between a star of type A0 and one of type K0 is about 0.5 mag. for this combination.

The method of determining the color of the nebulosity consisted

in measuring the brightnesses of the nebula through the pair of filters and applying corrections for the brightness of the sky background in the neighborhood. Two regions were taken for the brightness of the sky background, one at a distance of 110 minutes of arc west and 40 minutes north of Alcyone, and the other 25 minutes east and 80 minutes south of Alcyone. The region to the south was slightly more luminous, but we have used the average of both regions, since the accuracy for such faint objects is not very great. Eight regions within the nebulosity were measured, two about Maia and the remainder about Merope, mostly in a series of increasing distances south of the star. One region was measured three times and the variations in color were found to be rather large, about half of the total variation within the nebula.

There is an indication that the color of the nebulosity is bluer near the stars and becomes progressively redder at greater distances, but it will be necessary to have more observations before making a definitive statement. The average color is -0.07, with a probable error of ± 0.05, after the correction for atmospheric extinction has been made. Observations of the standard star 25 η Tauri (Alcyone) were made at the beginning and the end for the extinction factor. Its color is $+0.040$ mag. The colors of 20 c Tauri (Maia) and 23 d Tauri (Merope) were also determined and were found to be $+0.031$ and $+0.060$ mag., respectively. The mean of the three stars is $+0.044$ mag. It is seen that the color of the nebulosity is about 0.1 mag. bluer than that of the stars involved.

The two values of $\Delta_{\lambda_1} - \Delta_{\lambda_2}$, when reduced to the same interval of $\lambda_1 = 5000$ A and $\lambda_2 = 3900$ A, give:

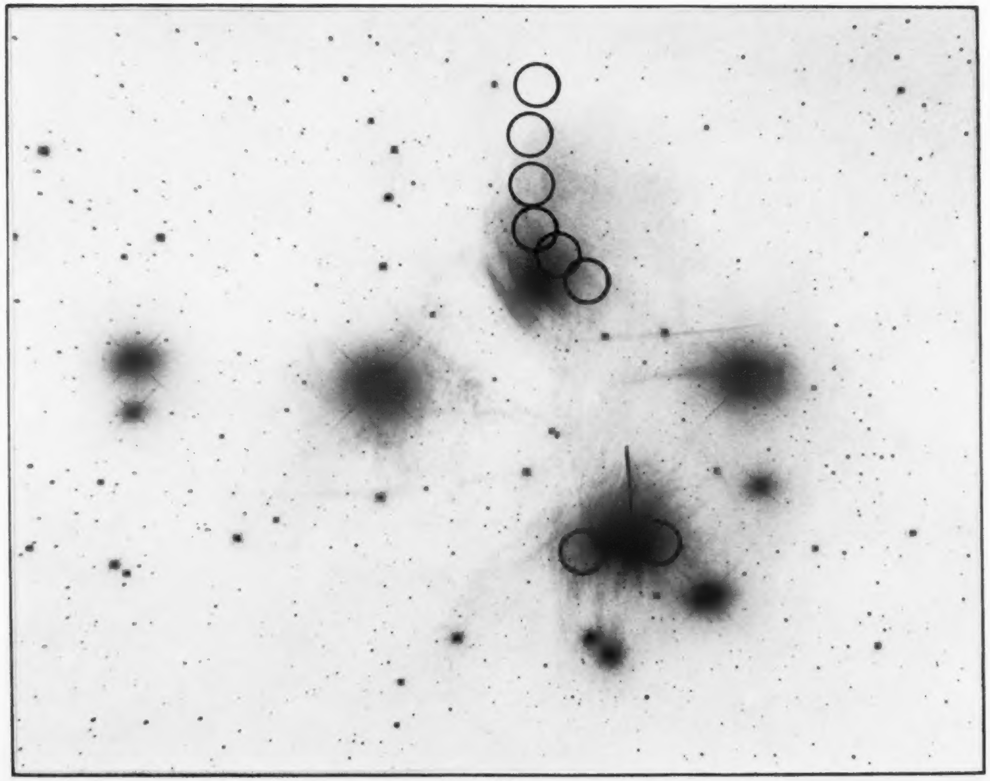
Spectrophotometric method	$\Delta_{\lambda\,5000} - \Delta_{\lambda\,3900} = +0.3$ m
Photo-electric method	$\Delta_{\lambda\,5000} - \Delta_{\lambda\,3900} = +0.2$
Mean	$\Delta_{\lambda\,5000} - \Delta_{\lambda\,3900} = +0.25$

Since the theory of Rayleigh scattering calls for a difference of 1.1 mag. for this interval, it seems certain that the light of the nebulosity surrounding these stars is not all due to Rayleigh scattering. There is, however, an indication that the light of the nebula is some-

PLATE VIII

S

E



W

N

Nebulae in the Pleiades (from a Plate Taken by G. W. Ritchey with the 24-Inch Reflector)

The straight line indicates the position of the spectrograph slit. Circles indicate positions of the receiver of the photometer.

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what bluer than that of the star, and it is therefore probable that the nebula consists of a mixture of particles of various sizes.[18]

Finally, we have examined a spectrogram of the variable nebula, N.G.C. 2261, obtained by Hubble in December, 1915,[19] with a 15° objective prism attached to the UV camera of the Yerkes Observatory. Hubble found the spectrum to be continuous. It was further investigated by V. M. Slipher,[20] who found that the spectra of the nebulosity and of the nucleus are the same, being similar to that of a nova. Our re-examination of the objective-prism spectrogram shows that there is no noticeable enhancement of the blue end of the spectrum of the nebula in comparison with the spectrum of the star. Hence, in this case, too, most of the light of the nebulosity must come from the reflection of starlight by larger particles.

In this discussion we have assumed that scattering is produced merely by free atoms or by solid particles. There is another type of scattering, that from free electrons, which is independent of the wave-length and which could produce an energy distribution similar to that observed by us in the nebula of the Pleiades. A. S. Eddington[21] has investigated this type of scattering from the point of view of the classical theory and has found[22] that for an ionized gas the scattering from free electrons is far more effective than the Rayleigh scattering. However, in a gaseous nebula the state of ionization must increase with the temperature of the exciting star, and we

[18] Hubble gives for the nebula in the Pleiades a color excess of zero (*op. cit.*, **56**, 404, 1922), and our result is not therefore contradictory to his. It would be interesting to investigate the energy distribution in the spectra of nebulae, the exciting stars of which have large color excesses. If the latter is caused by Rayleigh scattering, there must be an appreciable excess of violet light in the nebula. If this should be substantiated, we might conclude that the distribution of particles of different sizes is not the same in all nebulae, but that those of large color excess contain a greater proportion of small particles. As we have pointed out, observations of color excess alone are insufficient to establish this; small color excess may indicate one of two things: either the particles are so large that scattering is independent of λ or the density and the length of the light-path H are too small to produce a measurable amount of reddening.

[19] *Ibid.*, **44**, 190, 1916.

[20] *Lowell Observatory Bulletin*, No. 81, 1918.

[21] *The Internal Constitution of the Stars* (German ed.), p. 99, 1928.

[22] *Ibid.*, p. 485.

should therefore expect that nebulae with bright lines have stronger continuous spectra than those which show no bright lines. This is contrary to observation. Accordingly, it is even more difficult to attribute the luminosity of nebulae to electron scattering than to scattering from larger particles.[23]

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January 18, 1933

[23] This difficulty prompted Hubble to suggest that the origin of continuous nebular spectra may be due to a process of absorption and re-emission. The possibility of scattering from large particles was mentioned by Gerasimovič (*Zeitschrift für Astrophysik*, **4**, 279, 1932) in his theoretical discussion of Rayleigh scattering and abnormal stellar temperatures.

NOTE ON THE SURFACE BRIGHTNESSES OF ELLIPTICAL NEBULAE

BY PHILIP C. KEENAN

ABSTRACT

Mean *surface brightnesses* were computed for elliptical nebulae from the dimensions and total magnitudes given in the catalogue of Shapley and Miss Ames. The apparent increase in surface brightness with diminishing area can be attributed in part to the *Hubble effect* for threshold images, and in part to a *systematic trend in the computed areas*. The magnitude scale shows no evidence of large systematic variations.

The scarcity of good direct measures of the surface brightnesses of extra-galactic nebulae naturally suggests the use of mean values, computed from total magnitudes and measured dimensions, in making statistical studies. Thus, for example, E. F. Carpenter[1] has employed J. Holetschek's visual magnitudes and the dimensions given by E. P. Hubble to test the absorption of light in space by means of the variation of the average surface brightness of the nebulae with galactic latitude.

More accurate and complete data than have hitherto been available for investigations of this sort are provided by the recently published paper, "A Survey of the External Galaxies Brighter than the Thirteenth Magnitude," by H. Shapley and Miss A. Ames.[2] The total magnitudes in their catalogue were estimated by direct comparison with the focal images of stars on plates taken with cameras of very short focus. The lengths of the major and minor axes were compiled from the more important descriptive catalogues, supplemented where necessary by measurements on Harvard plates. The values furnished by Hubble were taken as standard and were given the greatest weight.

Inasmuch as practically all the nebulæ down to the given magnitude are included in this list, it seemed that an examination of the variation of surface brightness with apparent area would be of interest. In this preliminary study attention was confined to the elliptical nebulae, since they are much more uniform with respect to

[1] *Publications of the American Astronomical Society*, **7**, 25, 1931.

[2] *Harvard Annals*, **88**, No. 2, 43, 1932.

distribution of light than are the spirals with their extended arms. For yet greater homogeneity the material was subdivided according to the degree of elongation of the objects, following Hubble's classification.

In computing the surface brightness each nebula was considered as an ellipse of area $\pi\, ab/4$, where a and b are the axial dimensions given in the catalogue. The magnitude per square minute of arc was chosen as a convenient unit of surface brightness.

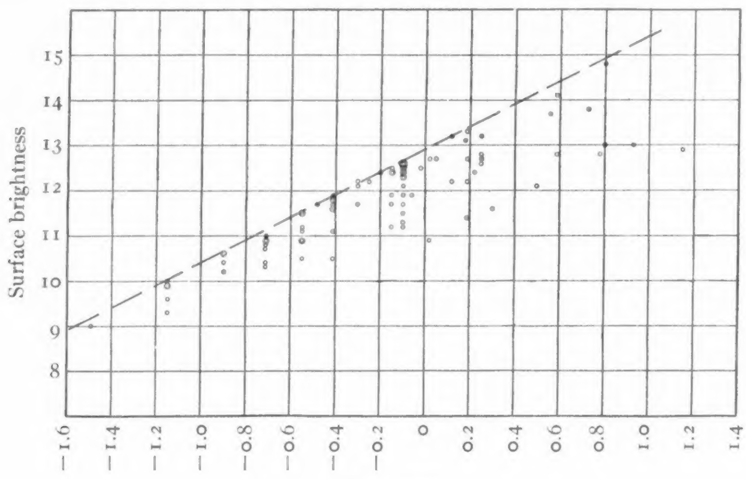

FIG. 1

The results for types $E_0 - E_1$, for which the data are most complete, are shown in Figure 1, where the circles represent the individual nebulae. A similar diagram constructed for the types $E_6 - E_8$ showed the same distribution, though the number of objects was much less. The apparent rise in surface brightness with decreasing log area is striking, but the sharpness with which the band of points is cut off on the upper side, as though bounded by a straight line with a slope different from that of the band itself, suggests the presence of systematic influences in the observational material or in the method of reduction.

Confirmation of this suspicion is furnished by Hubble's recent

discussion of the surface brightness of threshold images.[3] From his investigation of the relation between surface brightness and diameter of images just perceptible on his plates he concluded that, while the surface brightness is independent of the size of large images, the slope of the line for small diameters, less than 0.1 mm on his exposures, corresponds to the case in which the total luminosity, and not the surface brightness, reaches a certain threshold value. The data, in Hubble's words, "would indicate that the limiting magnitudes of plates having the same exposures are independent of the size of the threshold images. The observations show that the relation holds very closely for small images."

If the total brightness is to remain constant, the surface intensity must vary inversely with the area. For the equation of the line of limiting magnitude we have then:

$$\text{Surface brightness} = 2.5 \log \text{area} .$$

The images on the Harvard plates, taken with 2-inch Tessar lenses giving a scale of six hundred seconds to the millimeter, should follow this formula closely for objects having a maximum diameter less than about 1′. This point of transition will probably change according to the conditions of observation, but in general the larger nebulae might be expected to show a gradual falling-away from the straight line. On the assumption that the diameters of the focal images on the Harvard plates are proportional to the real diameters of the nebulae, the dotted line in Figure 1 was drawn with the computed slope and placed in such a way as to coincide with the faintest images at the center of the diagram. The accuracy with which it defines the limit of visibility of the nebulae throughout the observed range of size leaves little to be desired.

The Hubble effect is well illustrated by these results, and probably has some residual influence on images above the threshold, but it does not explain why no high values of surface brightness appear among the large nebulae. The observational factors which might influence the data as a whole in this way may be separated into two sorts:

1. *Error in the magnitude scale for nebulae.*—If a magnitude equa-

[3] *Astrophysical Journal*, **76**, 106, 1932.

tion exists it should be evident when surface brightness is plotted against total magnitude. Such a diagram (Fig. 2) was constructed for the $E_0 - E_1$ nebulae on the Harvard list, but the complete lack of any correlation between the plotted quantities eliminates this factor. The Harvard nebular magnitudes appear to be free from serious systematic errors. It is interesting to note that the maximum observed range in surface brightness amounts to 5 or 6 mag.

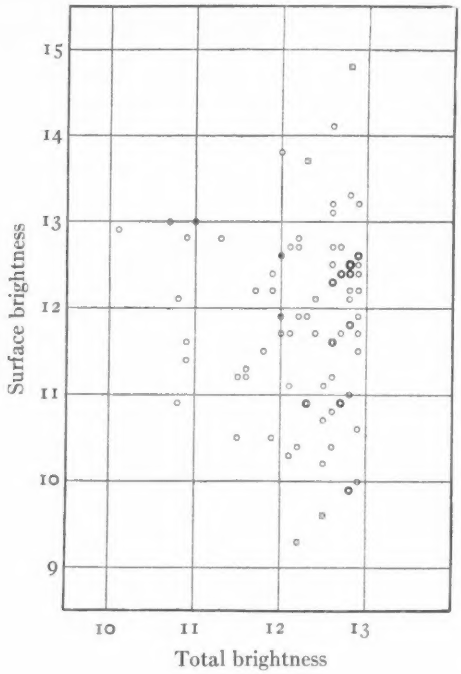

FIG. 2

2. *Error in the scale of dimensions.*—This possibility can be tested by comparison with data from independent sources. The visually estimated total magnitudes of Holetschek[4] are well adapted to this purpose by virtue of their freedom from photographic effects. Figure 3 was drawn analogously to Figure 1, use being made of the Harvard areas and of Holetschek's original estimates of magnitudes, without any corrections, since W. E. Bernheimer[5] has shown that Holetschek's scale coincides almost exactly with the Harvard system of magnitudes. In Figure 3 the points show the same systematic distribution that was found before, the slope of the band being only slightly different, though its upper side is naturally not as sharply bounded as in the case of the photographic observations.

Additional evidence is furnished by the measurements carried out by C. Wirtz on 98 nebulae in the Coma-Virgo cluster.[6] Wirtz's inte-

[4] *Annalen der K. K. Universitäts-Sternwarte in Wien*, **20**, 39, 1907.

[5] *Lund Observatory Circulars*, No. 5, 85, 1932; No. 6, 107, 1932.

[6] *Publication der Sternwarte in Kiel*, **15**, 35, 1927.

grated magnitudes agree very closely with those of Shapley and Miss Ames, since he adjusted the zero point of his scale by comparison with the earlier lists published at Harvard, and proceeded analogously to them, except that photometric readings with a Rosenberg microphotometer took the place of visual estimates of the images. Thus the trend of the surface brightnesses computed from his magnitudes and from the Harvard areas is equivalent to that of Figure 1, but when the areas are computed from the dimensions measured by

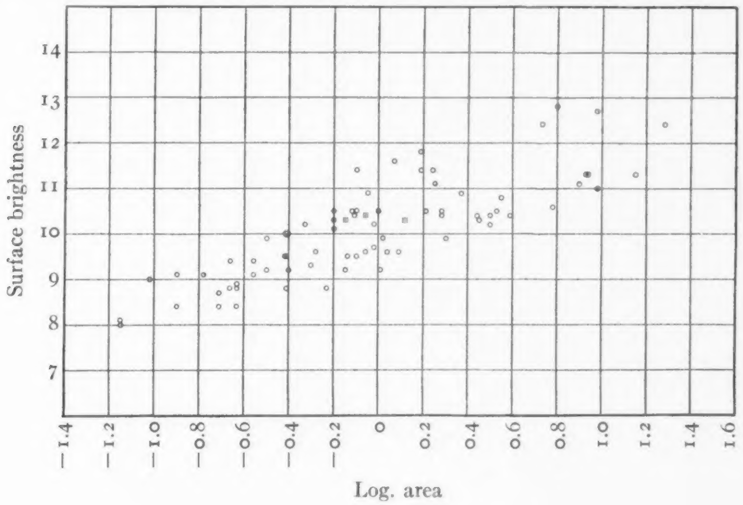

FIG. 3

Wirtz himself, the surface brightness appears to be practically independent of the area for the rather short range covered.

It is of further interest to observe that these computed mean values are systematically different from the direct measures of surface brightness which Wirtz made on the same plates. The latter increases with the log diameter of the elliptical nebulae, as may be seen from Figure 4 of his paper. The discrepancy cannot be ascribed entirely to the faults of the computed values. If the direct determination of surface brightness is carried out by measuring a more or less constant area near the center of each nebula, as was done by Wirtz, it is obvious that the result will apply chiefly to the nucleus in the case of the larger nebulae, whereas the brightness of the small

objects will appear to be less because a greater proportion of the faint outlying parts will be included. This effect was certainly present in the earlier visual measurements made by Wirtz on a large number of nebulae,[7] as he remarked at the time, and may explain entirely the even greater increase in surface brightness with log diameter indicated by those observations.[8]

Hubble has shown that the intensity of light at any distance r from the center of an elliptical nebula varies as log $(r/a+1)$, where a is a parameter which varies considerably from one nebula to another.[9] When the value of a has been found, it is possible to compute the surface brightness as well as the total magnitude from the intensity at the center, but this individual treatment is too laborious for the accumulation of much statistically useful data. However, Hubble found also that the central luminosity varies as the square of the diameter of the image for a given exposure time, the fifteen nebulae investigated following this relation closely. If sufficient material were available to permit the variations in a to be neglected, this law would indicate constant surface brightness, but in any case it suggests that the relative values of the surface brightnesses can be fairly well determined by comparing measures made on plates taken with approximately the same time of exposure.

The results of these comparisons imply strongly that the distribution shown in Figure 1 is due largely to a systematic trend in the observed dimensions, in the sense that there is a tendency to measure a greater proportion of the true area of a large nebula than of a small one. Such a tendency has been noted before by Lundmark[10] and probably by others, and may well be a general characteristic of the principal published lists, most of which were drawn upon in the preparation of the Harvard catalogue. The importance of this catalogue for statistical work is emphasized by the confirmation found here of the consistency of the magnitude scale, but care must be taken in the application of the table of dimensions. We are not

[7] *Meddelanden från Lunds Astronomiska Observatorium* (2d ser.), No. 29, 1923.

[8] *Probleme der Astronomie: Festschrift für Seeliger*, p. 69, 1924.

[9] *Op. cit.*, **71**, 231, 1930.

[10] *Meddelanden från Astronomiska Observatorium Upsala*, No. 22, 6, 1927; No. 41, 11, 1928.

yet in a position to draw any valid conclusions with respect to the true distribution of surface brightness with size of the nebulae.

The problem of the measurement of dimensions requires further attention. Lundmark[11] compared the measures of diameters made by H. D. Curtis[12] on Lick plates, and repeated by himself, with those carried out by K. Reinmuth at Heidelberg,[13] and found that the agreement was good. This he took to indicate that the scale of the plates has little effect on the measurements. The agreement between these two catalogues is undoubtedly fair, but when they are compared with the Harvard and Wirtz magnitudes large discrepancies appear, as is evident from Table I, a short table of areas in square

TABLE I

AREAS OF ELLIPTICAL NEBULAE

N.G.C.	Catalogue			
	Harvard	Wirtz	Curtis	Reinmuth
4270	0.76	0.20	0.23	0.55
4374	5.9	1.8	0.79	2.8
4472	14.1	2.8	3.1	9.4
4473	3.6	0.85	....	1.5
4474	0.82	1.2	....	0.40
4478	0.94	0.64	0.20	0.28
4486	8.6	2.0	3.1	8.0
4564	0.94	1.0	....	0.57
4649	9.5	2.6	3.1	5.7
4660	0.39	0.50	0.31	0.56

minutes of arc for objects common to the four lists, along with a few other nearby nebulae. Large constant differences are to be expected from the varying effective exposures of the several series of plates, but even in the small region of the sky represented in the table the differences actually found are inconsistent. Thus N.G.C. 4472 is measured as five times as large in the Harvard catalogue as in that of Wirtz, but 4474, 4564, and 4660 appear larger in the latter. The correspondence between the Harvard and Heidelberg values is closer, but the former indicates that N.G.C. 4478 is twice as large as 4660, while Reinmuth finds the inverse ratio between the two.

[11] *Lund Observatory Circular*, No. 3, 54, 1931.

[12] *Publications of the Lick Observatory*, **13**, 1, 1918.

[13] *Veröffentlichungen der Badischen Sternwarte zu Heidelberg*, **9**, 1, 1926.

A detailed comparison of the various catalogues is beyond the scope of this note, but these few examples are sufficient to draw attention to the importance of including accurate and homogeneous measurements of dimensions in the co-operative survey of extragalactic nebulae, now being undertaken by several observatories according to the general plan of Hubble.

YERKES OBSERVATORY
December 1932

SOME EFFECTS OF CHANGES IN STELLAR TEMPERATURES AND ABSOLUTE MAGNITUDES

By W. W. MORGAN

ABSTRACT

The behavior of some of the more abundant metals is traced through a representative temperature series of giants and dwarfs. *Mg* II, *Fe* II, *Cr* II, and *Ti* II show "null effects" on the high-temperature side of their maxima. In the case of *Mg* II there is a null effect from O9 through B5, for *Fe* II and *Cr* II at B8, and for *Ti* II at A0 and A2. There is a pronounced absolute-magnitude effect for all four elements on passing to lower temperatures. In the dwarfs the maximum for *Mg* II, *Fe* II, and *Cr* II is at A0–A2. The maximum is sharper for *Mg* II than for *Fe* II or *Cr* II. In the case of *Ti* II the lines in the dwarf sequence remain sensibly unchanged in intensity from A2 through K5. In the giants the maximum for *Mg* II is between A2 and F5 while for *Fe* II, *Cr* II, and *Ti* II it is at F5. *Ba* II shows no null effect corresponding to the other elements; it is weakened in giants up to between F5 and F8, after which it is strengthened. *Sr* II and *Fe* I behave like *Ba* II. The relative numbers of atoms contributing to line intensities in giants and dwarfs are found by means of the Russell-Adams calibration of the Rowland intensity scale. At spectral type F5, where the maximum absolute-magnitude effect takes place for *Mg* II, *Fe* II, *Cr* II, and *Ti* II, there are around twenty times as many atoms contributing to the lines of each of these elements in supergiants as in dwarfs.

In earlier determinations of the position of maximum intensity of many of the common elements in the stellar temperature scale, stars of all absolute magnitudes were grouped together and possible effects of pressure on the intensities of the lines were in general disregarded. Because of the lack of exact information concerning the behavior of even the common metals, it seemed worth while to investigate the differences due to changes in temperature and pressure in standard spectra. This study is intended merely to enumerate the differences observed and was not undertaken as a test of theoretical predictions of absolute magnitude and temperature effects.

The stars selected for the study were temperature sequences of supergiants and dwarfs. The most luminous stars for which spectra were available were chosen for the giants, and a group of dwarfs which occupy as near an average position as possible in the Russell-Hertzsprung diagram was selected as standard of low luminosity. For this reason the absolute magnitudes of the dwarfs decrease from $+1.3$ for Sirius at A0 to $+8.0$ for 61 Cygni at K5. As the main

sequence joins the giant sequence at about A0 the stars of type B are not strictly comparable with those of later type. The difference in absolute magnitude between the pairs of giants and dwarfs in types O9–B5 inclusive is less than for those of later type. From a consideration of the Stark effect for helium, Struve found that the largest differences may be more than 3 mag. Giants and dwarfs of types earlier than B8 were taken from Struve's groups.[1] Those of types B8 and later consist of *c*-stars for the giant sequence and intrinsically faint stars of large trigonometric parallaxes for the dwarfs. In determining the absolute magnitudes of the standard stars, trigonometric and dynamical parallaxes alone were used.

The plates of the stars of classes earlier than A0 were of one-prism dispersion and had been obtained on the fine-grain Eastman Process emulsion. The other spectra were of three-prism dispersion

TABLE I

λ	*R*	λ	*R*
4558	3	4494	6
4563	4	4528	8
4501	5	4404	10
		4340	20

and had been taken on Eastman 40 plates. The scale of the one-prism plates is 30 A per millimeter, and of the three-prism spectra 10 A per millimeter at λ 4500. The plates were obtained with the Bruce spectrograph attached to the 40-inch telescope of the Yerkes Observatory.

Strong unblended lines of the elements *Mg* II, *Ti* II, *Fe* I, *Fe* II, *Ba* II, and *Cr* II were selected. By means of standard solar spectra taken with the same instrument and the same kinds of plates the stellar intensities of these lines were estimated on the Rowland solar scale. The lines which were used as standards of intensity are given in Table I. As has been pointed out by Russell, Adams, and Miss Moore,[2] the Rowland scale is rather ragged and errors of estimation are apparent on inspection of solar spectra. For example, the line *Ba* II 4554 is given as of intensity 8 in Rowland's estimates, although it is much weaker than λ 4528 (8) and is no stronger than λ 4494 (6).

[1] *Astrophysical Journal*, **74**, 250, 1931.

[2] *Ibid.*, **68**, 277, 1928.

Another example of occasional irregularities is the line λ 4383 (15), which is actually stronger than $H\gamma$, λ 4340 (20).

When the lines are estimated on Rowland's scale it is possible to deduce the relative number of atoms effective in absorbing the different lines by means of the calibration of the solar spectrum intensities made by the three authors aforementioned.[3] As the calibration is based on the mean of a large number of lines, there may be some systematic error introduced by singling out a few lines to be used as standards; such an error, however, must be small, as the standard lines are close to the average strength for lines of their tabular intensities, and form a consistent scale among themselves. Lines of intensity 1 are very close to the limit of visibility on three-prism Eastman 40 plates. The differences in intensity of lines at intervals of 1 unit on Rowland's scale are easily seen in the range of intensities 3–10, and it seems unlikely that an error of a whole unit will result if estimates are made on several well-exposed plates of a star which has lines of good quality. Above intensity 10 the scale becomes more compressed and only two intermediate steps have been included between 10 and 20. As the lines occur in a narrow range of spectrum, no corrections for differences in dispersion have been made. A correction was made to the values of log N for the Rowland intensities of the standard lines to reduce Russell's factor B to unity.

The lines selected for the study were *Mg* II 4481, *Ti* II 4501, *Fe* I 4376 and 4528, *Fe* II 4508, *Cr* II 4558, and *Ba* II 4554. Qualitative observations of the behavior of *Sr* II 4215 were also made but no intensity estimates are included. The intensity of *Mg* II 4481 was estimated in classes O9–F8, *Fe* II 4508 was observed from B8–K0, and the other lines were considered in the range A0–K5. The observed data are given in Table II. The columns give (1) the name of the star; (2) the H.D. spectral type (except for Maia, which is considered to be of type B8 although H.D. gives B5, and for ι Herculis, which is put in the B5 group by Struve); (3) the classification as giant or dwarf (the giants of types B8 and later are all supergiants); (4)–(10) the estimated intensities on the Rowland scale for

[3] *Ibid.*, p. 271.

the standard lines; (11)–(17) relative values of log N obtained for the same lines by the Russell-Adams-Moore calibration.

Some of the results of Table II are plotted in Figure 1. *Mg* II 4481 (excitation potential 8.8 volts) shows a null effect from O9 to B5. In this range there is no observable difference in intensity between giants and dwarfs of the same spectral class. The line increases gradually and regularly from O9 to A2 in the giants. The intensity at F5 is the same as at A2 but there is a definite weakening at F8. *Mg* II is unobservable in stars later than F8 because of

TABLE II

Star	Sp. H.D.	M	Intensity (Rowland) 4481	4501	4508	4376	4528	4554	4558	log N 4481	4501	4508	4376	4528	4554	4558
λ Orionis	O9	g	4	....	....	....	....	....	....	2.1	....	....	....	....	....	....
10 Lacertae	O9	d	4	....	....	....	....	....	....	2.1	....	....	....	....	....	....
9 Camelopardalis	B0	g	6	....	....	....	....	....	....	2.9	....	....	....	....	....	....
τ Scorpii	B0	d	6	....	....	....	....	....	....	2.9	....	....	....	....	....	....
ζ Persei	B1	g	6	....	....	....	....	....	....	2.9	....	....	....	....	....	....
β Cephei	B1	d	6	....	....	....	....	....	....	2.9	....	....	....	....	....	....
χ^2 Orionis	B2p	g	7	....	....	....	....	....	....	3.1	....	....	....	....	....	....
γ Pegasi	B2	d	7	....	....	....	....	....	....	3.1	....	....	....	....	....	....
ε Cassiopeiae	B3	d	8	....	....	....	....	....	....	3.2	....	....	....	....	....	....
67 Ophiuchi	B5p	g	9	....	....	....	....	....	....	3.3	....	....	....	....	....	....
ι Herculis	[B5]	d	9	....	....	....	....	....	....	3.3	....	....	....	....	....	....
β Orionis	B8	g	10	....	3	....	....	....	3	3.4	....	1.6	....	....	....	1.6
20 Tauri	[B8]	d	8	....	3	....	....	....	3	3.2	....	1.6	....	....	....	1.6
η Leonis	A0p	g	15	4	8	....	....	....	8	3.8	2.1	3.2	....	....	....	3.2
α Canis Majoris	A0	d	10	4	6	1:	1:	3	5	3.4	2.1	2.9	0.6:	0.6:	1.6	2.5
α Cygni	A2p	g	20	6	9	....	....	....	8	4.0	2.9	3.3	....	....	....	3.2
μ Orionis	A2	d	10	6	6	3	5	6	5	3.4	2.9	2.9	1.6	2.5	2.9	2.5
γ Virginis (S)	F0	d	8	5	5	4	6	6	4	3.2	2.5	2.5	2.1	2.9	2.9	2.1
ε Aurigae	F5p	g	20	20	20	....	4	3	10	4.0	4.0	4.0	....	2.1	1.6	3.4
γ Cygni	F8p	g	10	15	10	8	9	10	9	3.4	3.8	3.4	3.2	3.3	3.4	3.3
Sun	G0	d	1	5	4	6	8	6	3	0.6	2.5	2.1	2.9	3.2	2.9	1.6
ε Pegasi	K0	g	....	10	6	15	15	20	7	....	3.4	2.9	3.8	3.8	4.0	3.1
61 Cygni (br)	K5	d	....	5	< 3:	*	10	8	2:	....	2.5	<1.6:	*	3.4	3.2	1.1:

* Plates of 61 Cygni do not extend this far into violet.

blends. The first apparent absolute-magnitude effect occurs at B8 where the line is definitely stronger in Rigel than in Maia. There seems to be a slight depression in the dwarf-curve at B8. λ 4481 reaches its maximum intensity at A0 and A2 in the dwarf sequence, after which it diminishes rather rapidly until it disappears at about G0. There is a definite shifting of the maximum to higher temperatures on passing from giants to dwarfs. The greatest absolute-magnitude effect is at F5. At this point there are about thirty times as many atoms effective in absorbing the line in the supergiant as in the dwarf.

The line *Fe* II 4508 (E.P. = 2.8 volts) is first seen at B8, where it is of the same intensity in the dwarf Maia and the supergiant Rigel.

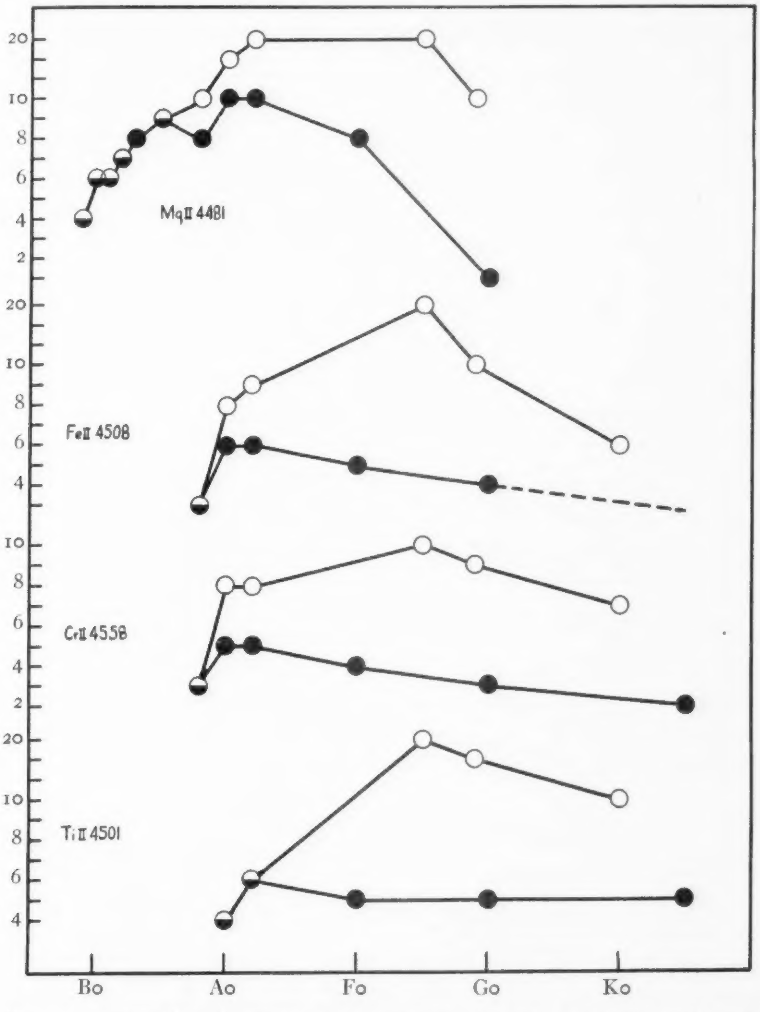

FIG. 1*a*.—Intensities of some of the common elements in giants and dwarfs. Abscissae are H.D. spectral types; ordinates are intensities on the Rowland solar scale. Open circles are giants; closed circles are dwarfs.

There is a definite absolute-magnitude effect at Ao; the line is stronger in the *c*-star η Leonis than in Sirius. This absolute-magnitude effect increases in amount to F5, where there are roughly twenty times as

many atoms absorbing the line in the supergiant as in the dwarf. After this point the absolute-magnitude effect decreases. In the dwarfs the line is strongest at A0 and A2 and declines very gradually in intensity until it is lost in a blend at about type K5. It is never a

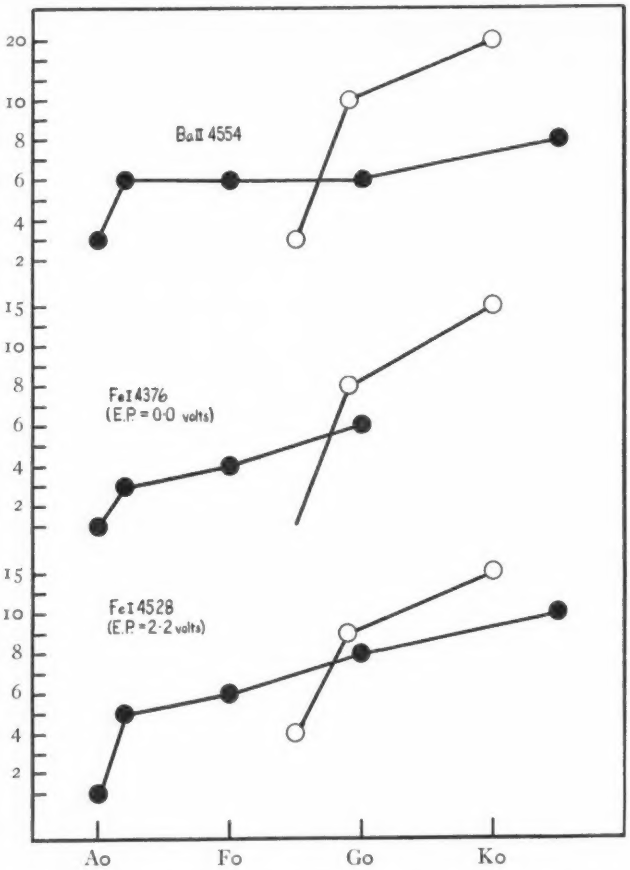

FIG. 1*b*.—Intensities of some of the common elements in giants and dwarfs. The explanation is similar to Fig. 1*a*.

very strong line in the dwarf sequence, and it seems safe to say that the enhancement observed in certain stars is due mostly to high absolute magnitude. In the giant sequence the line comes to a maximum at type F5. The finding of maxima for *Fe* II between the positions of maxima for giants and dwarfs is probably due to mixtures

of stars of different absolute magnitudes. It is also probable that the difficulty experienced by some observers in finding a maximum at all for *Fe* II and *Ti* II was due to failure to separate the large absolute-magnitude effects from the relatively small effects of changes in temperature.

Although *Ti* II 4501 (E.P. = 1.1 volts) is present in some B9 stars, no satisfactory dwarf of this spectral type was available, and the first observed intensity given is for type A0. At both A0 and A2 there is a null effect. The line is noticeably stronger at A2 than at A0. In the giant sequence it behaves quite similarly to *Fe* II 4508 and comes to a maximum at F5. The greatest absolute-magnitude effect is also at this place, with about twenty times as many atoms absorbing the line in a supergiant as in a dwarf. The line decreases in intensity in the giants after F5 until it is no stronger in Betelgeuse (Ma) than in 61 Cygni (K5), although the difference in absolute magnitude is over 10 mag. In the dwarf sequence there is very little change in intensity from A0 to K5. The line is as strong in 61 Cygni (K5) as in γ Virginis (F0). The intensity of *Ti* II in this spectral range seems to be entirely insensitive to differences in temperature and very sensitive to differences in pressure. If a maximum can be said to be present in the dwarf sequence, it is probably at type A2. The "null effect" observed for *Mg* II at O9–B5 does not have the same weight as that found for *Fe* II at B8 and *Ti* II at A0 and A2 where the range in absolute magnitude is greater. There can be no doubt, however, that *Mg* II is insensitive to changes in absolute magnitude in stars of type B5 and earlier.

Cr II 4558 (E.P. = 4.1 volts) behaves quite similarly to *Fe* II 4508. There is a null effect at B8, after which there is a marked enhancement of the line in the supergiants. The maximum in the giant sequence is at F5; in the dwarfs it is at A0–A2. The line is unobservable because of blends and weakness after K0. Throughout the range considered it is slightly weaker systematically than *Fe* II 4508 and *Ti* II 4501. The anomalous intensity of *Cr* I and *Cr* II in certain A dwarfs will not be considered here.

The behavior of the ultimate line of *Ba* II at λ 4554 is quite different from the lines of *Fe* II, *Ti* II, and *Cr* II. It is first seen in dwarfs at A0 and becomes considerably stronger by A2. It then remains

constant in intensity through G0 after which it becomes slightly stronger. The line is absent altogether from the supergiants until spectral type F5 is reached. It is quite faint at this place and is of the same intensity as in the A0 dwarf Sirius. Between F5 and F8, however, the sign of the absolute-magnitude effect changes and the line is decidedly strengthened in γ Cygni as compared with dwarfs of the same spectral type. A considerable absolute-magnitude effect in the same direction continues at least through K5.

Although the strong ultimate *Sr* II doublet is out of the range of the three-prism plates used for the other elements, the behavior of λ 4215 was also investigated qualitatively on one-prism plates. The absolute-magnitude effects are exactly like those of *Ba* II. The line is greatly weakened in supergiants of types A0 and A2, is considerably weakened at F5, and the sign of the effect changes between F5 and F8. It is stronger in the supergiant γ Cygni than in the F8 dwarf θ Ursae Majoris, and after this point is much enhanced in giants. It is obvious from an examination of A-type supergiants and dwarfs that in spectra earlier than F5 the strength of *Sr* II is a manifestation of intrinsically faint, not highly luminous, stars.

In the case of *Fe* I two lines of different excitation potential were observed. The excitation potentials were 0.0 and 2.2 volts. Little difference in the behavior of the two lines could be detected. Both behaved quite similarly to *Ba* II. They are at the limit of visibility at A0 in dwarfs and gradually increase in intensity along the main sequence. The line λ 4528 first appears at F5 in supergiants, at which place it is weaker than in dwarfs. The sign of the absolute-magnitude effect changes between F5 and F8, and both lines are enhanced in giants of later spectral types.

The changes which have been described can be traced on the reproductions of the standard spectra given in Plates IX and X.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
December 28, 1932

NOTE ADDED TO PROOF.—It should, perhaps, be mentioned that the description of the strength of certain lines in a particular spectral type refers entirely to the spectrum of the star taken as standard of that type.

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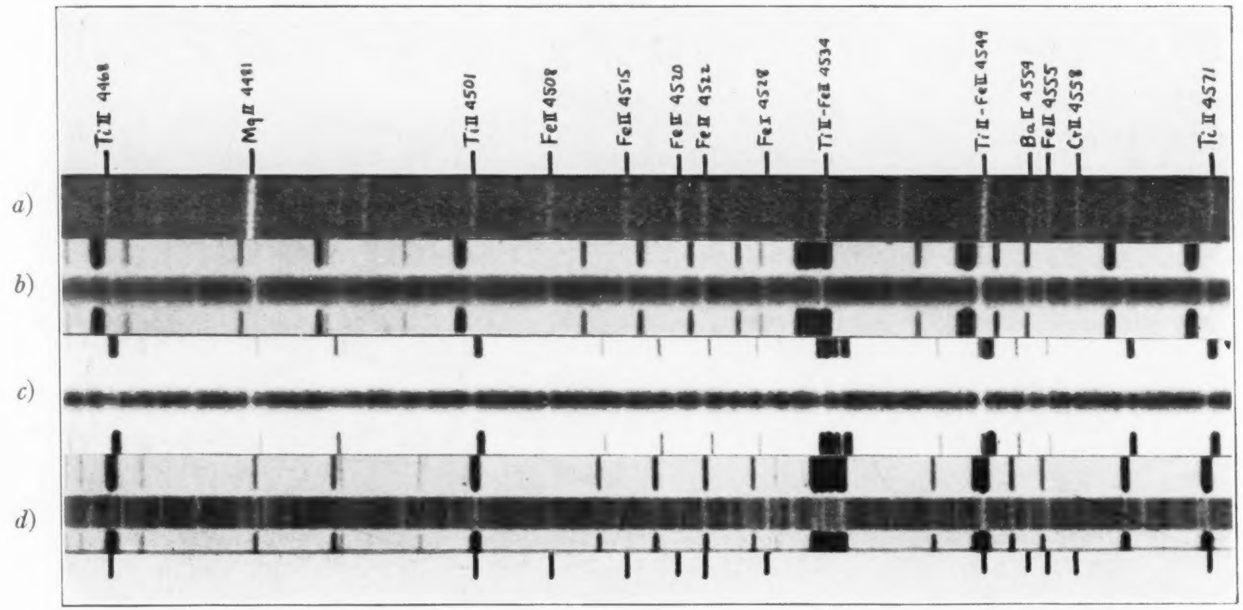
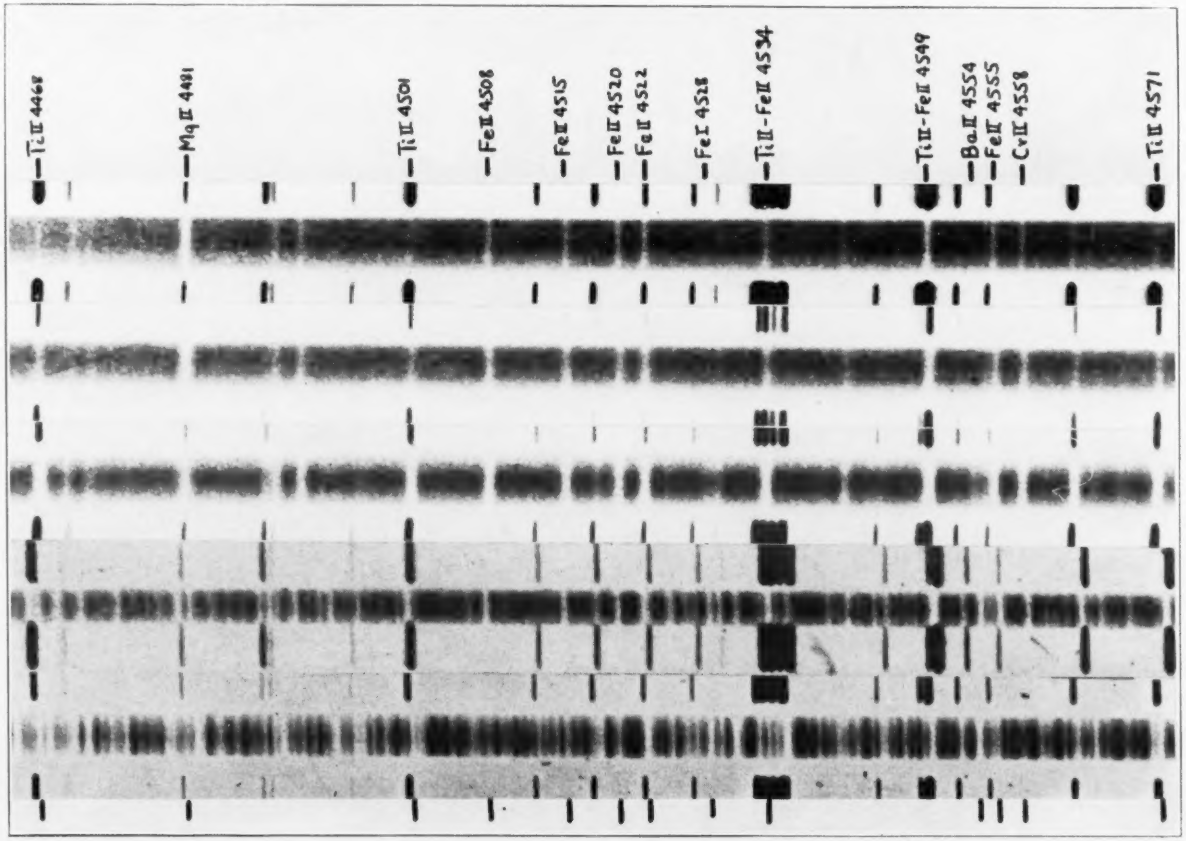
PLATE IX



SPECTRA OF STANDARD DWARFS: (*a*) α CANIS MAJORIS, A0; (*b*) μ ORIONIS, A2; (*c*) γ VIRGINIS (S), F0; (*d*) SUN, G0

The behavior of the lines with decreasing temperature can be followed and differences in intensity due to changes in absolute magnitude can be seen by comparing the dwarfs in Plate IX with the supergiants in Plate X.

PLATE X



a)
b)
c)
d)
e)

SPECTRA OF STANDARD SUPERGIANTS: (*a*) η LEONIS, A0; (*b*) α CYGNI, A2; (*c*) ϵ AURIGAE, F5; (*d*) γ CYGNI, F8; (*e*) ϵ PEGASI, K0

The changes of the lines with temperature are shown, and the absolute-magnitude effects can be seen by comparing with the dwarfs shown in Plate IX.

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NOTES

ABSORPTION LINES OF *N* V IN STELLAR SPECTRA

ABSTRACT

The lines of four-times ionized nitrogen require greater excitation than any others hitherto identified in stellar spectra. They have been observed, as absorption lines, in several Wolf-Rayet stars.

The spectrum of *N* v has recently been analyzed by B. Edlén, who has pointed out[1] the identity of the pair of bright Wolf-Rayet lines at 4620, 4604, observed by C. S. Beals[2] in H.D. 187282, with lines of this atom.

The same pair of lines has been observed in absorption in the spectra of a number of southern Wolf-Rayet stars,[3] notably H.D. 92740, 93131, and 151932. The general level of excitation of these spectra, and the occurrence, in the associated bright-line spectrum, of lines of nitrogen in lower stages of ionization (*N* III and *N* IV) leave no doubt as to the identifications.

The most highly excited absorption line previously known in stellar spectra was 4658.41 of *C* IV, an atom with ionization potential 64 volts.[4] Successive ionization potentials of nitrogen are: (*N* I) 14.48 volts; (*N* II) 29.47 volts; (*N* III) 47.40 volts; (*N* IV) 77 volts; (*N* V) 97.43 volts. The occurrence of a line corresponding to so high an ionization potential indicates[3] that the Wolf-Rayet stars may have surface temperatures greater than 80,000°.

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March 24, 1933

[1] *Arkiv för Matematik, Astronomi och Fysik*, **22**, B11, 1931.

[2] *Publications of the Dominion Astrophysical Observatory*, **4**, No. 17, 1930.

[3] C. H. Payne (in press).

[4] J. E. Mack, P. Swings, and O. Struve, *Astrophysical Journal*, **76**, 77, 1932.

REVIEWS

Report on Band Spectra of Diatomic Molecules. By W. JEVONS. London: Physical Society, 1932. Pp. 308. Pls. IV. Unbound, 17*s.* 6*d.*; bound, 20*s.* 6*d.*

The theory of band spectra has not received the attention from astrophysicists that it deserves, owing principally to the fact that investigators of these spectra have written almost exclusively for workers in their own field and have made only a few attempts to publish books and reports intended for workers in related sciences. Textbooks dealing with all aspects of atomic spectra from the most elementary to the most abstruse are available, but clear, concise, up-to-date expositions on molecular spectra have been relatively infrequent. With the publication of this *Report on Band Spectra* by W. Jevons, the astrophysicist has a convenient source of practical information on any subject connected with band spectra that is likely to arise in his work.

The author gives just enough detail concerning the theory of the different topics discussed to enable the reader to grasp the subject in its entirety without fear of losing sight of the forest for the trees. He gives the more important working equations resulting from the theory, and their application is fully explained by means of numerical examples, tables, and diagrams. For instance, in the chapter on the isotope effect, there is first a brief statement of the general nature of the isotopic displacements in band spectra. The vibrational and rotational isotope effects are taken up in detail and exact and approximate expressions given for the amount of the shift. Next the actual effects observed in different types of molecules at different temperatures are considered. Then follow several paragraphs of special astrophysical interest, such as intensities in bands of isotopic molecules, relative intensities and relative abundance of isotopes, and discovery of rare isotopes by the band spectra method.

The *Report* contains many tables and copious references which should be invaluable to spectroscopists. Only a few typographical errors were detected and these are of a trifling nature.

R. S. RICHARDSON